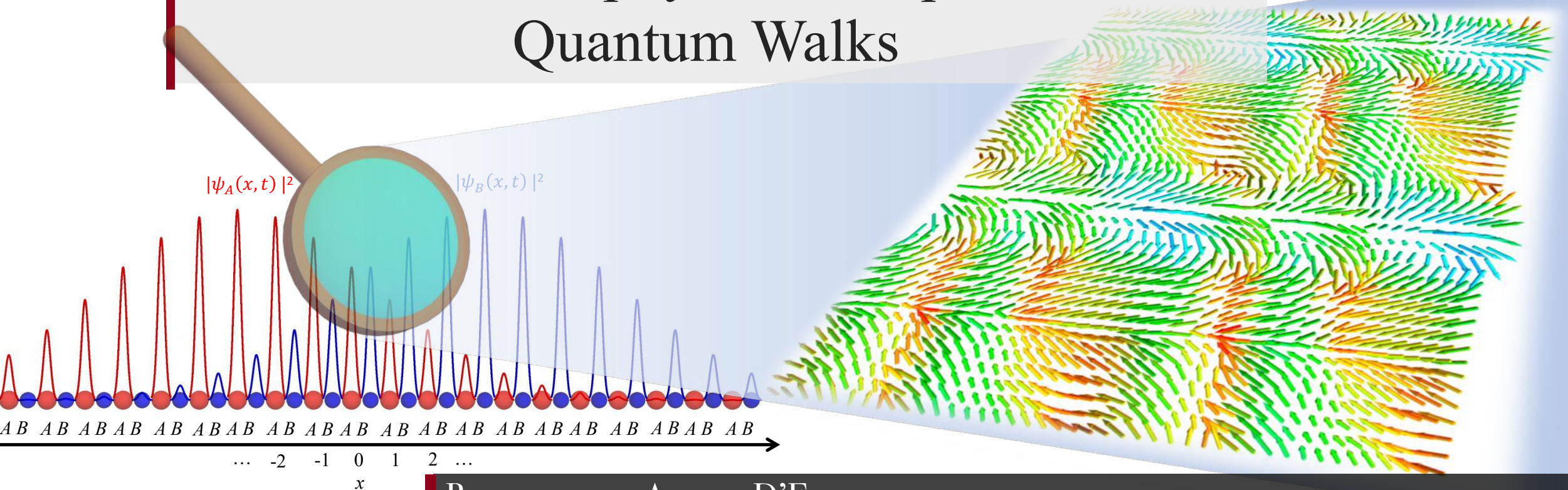
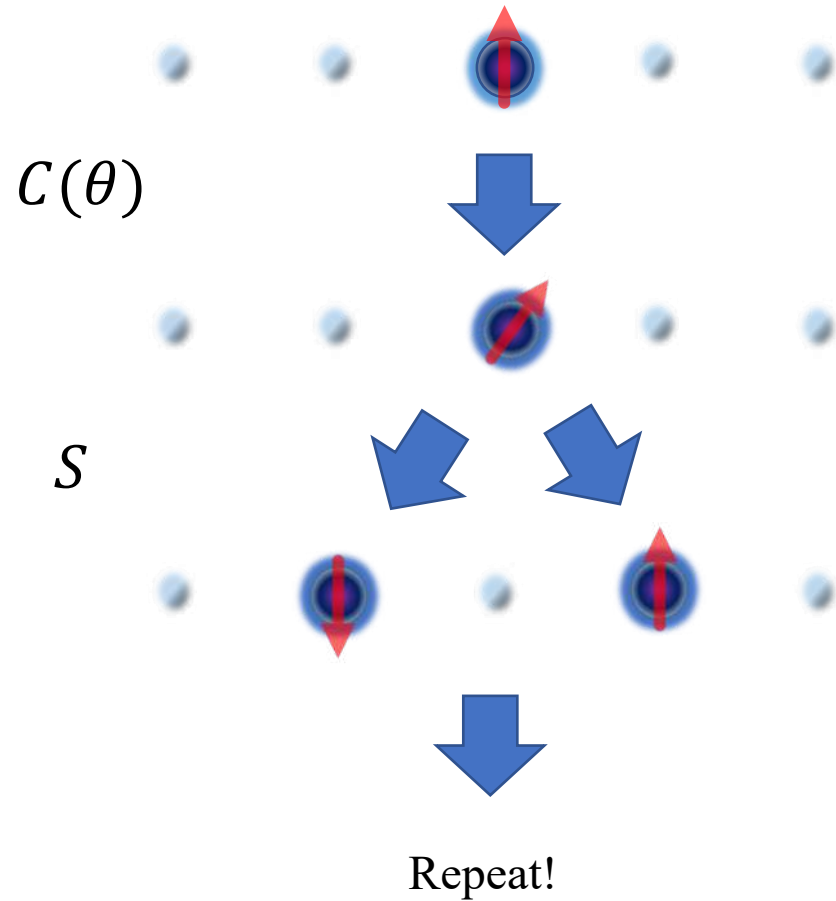


Understanding topology and geometry in solid-state physics with photonic Quantum Walks



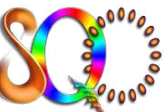
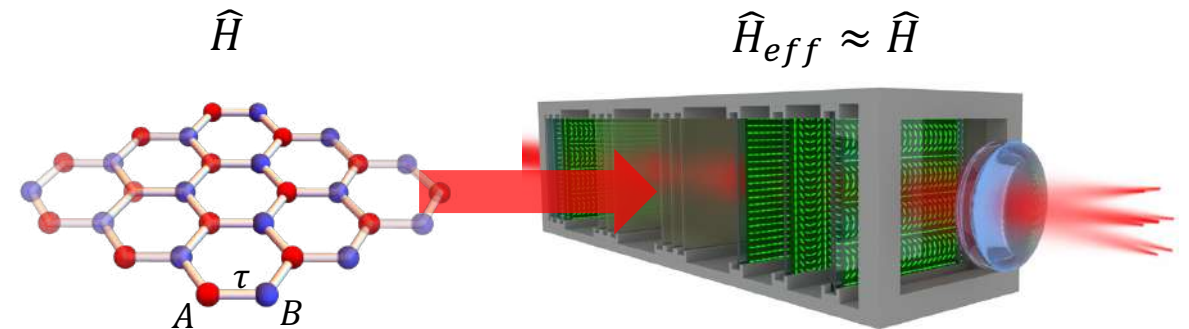
PRESENTED BY: ALESSIO D'ERRICO

Quantum Walks as iterated Unitary processes



$$U = S \cdot C(\theta); |\psi(t)\rangle = U^t |\psi(0)\rangle$$

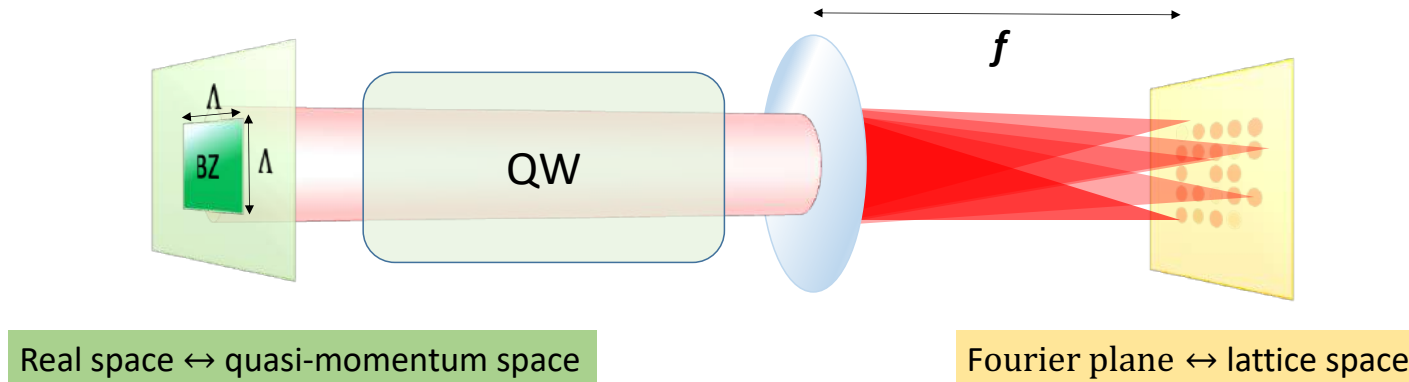
$$U^t = \text{Exp} \left(-\frac{iH_{eff}}{\hbar} t \right)$$



Quantum Walks as iterated Unitary processes

$$U^t = \int_{-\pi}^{\pi} \mathcal{U}^t(q) \otimes |q\rangle\langle q| \frac{dq}{2\pi}$$

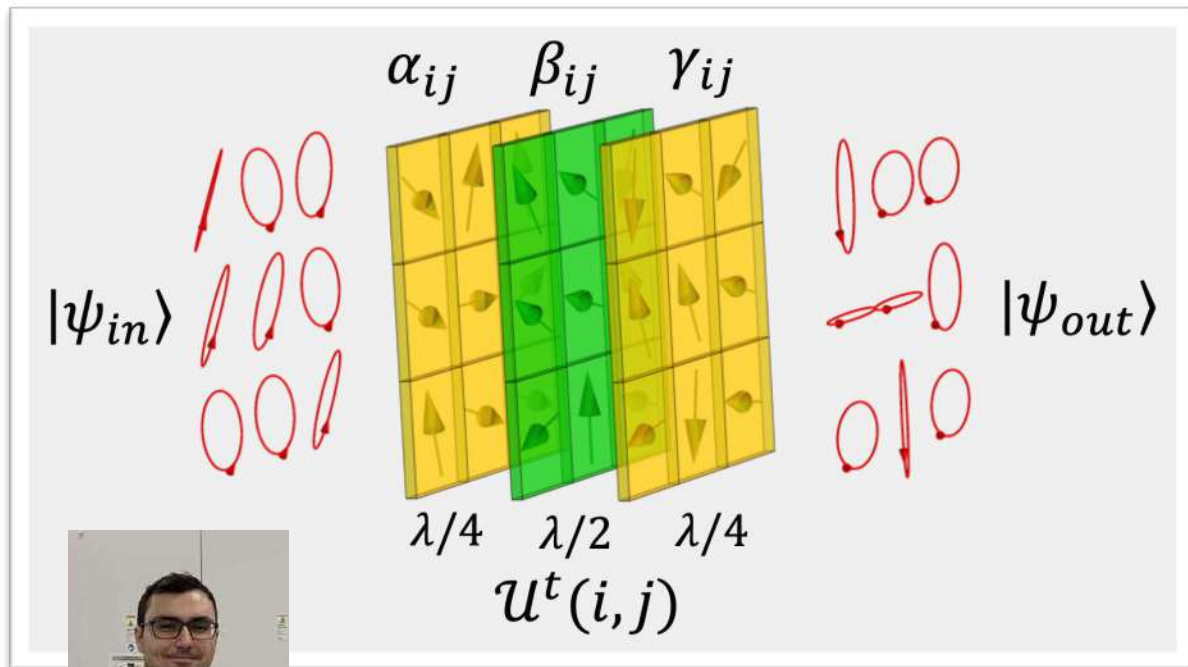
$$|q\rangle := \sum_X e^{iqX} |X\rangle$$



Customizing space dependent unitaries

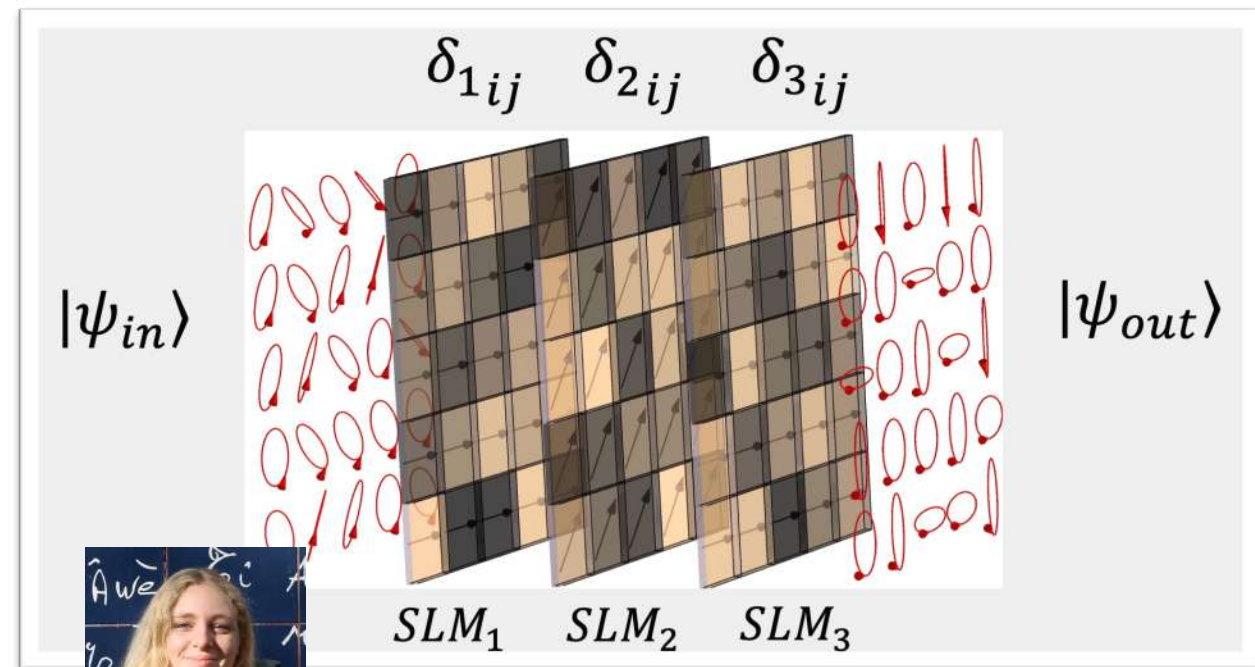
Liquid crystal metasurfaces

- Fixed retardation
- Variable optical axis



Spatial light modulators

- Variable retardation
- Fixed optical axis

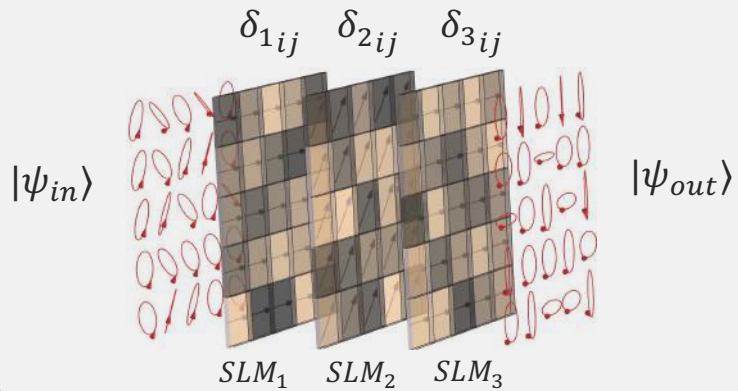


P_28 Maria G. Ammendola

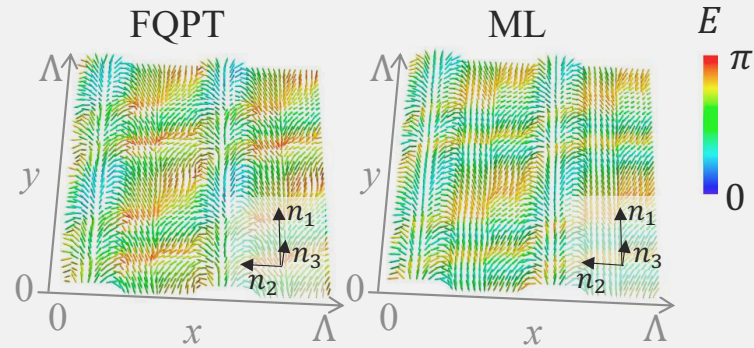


QW with structured light

Reconfigurable photonic processors

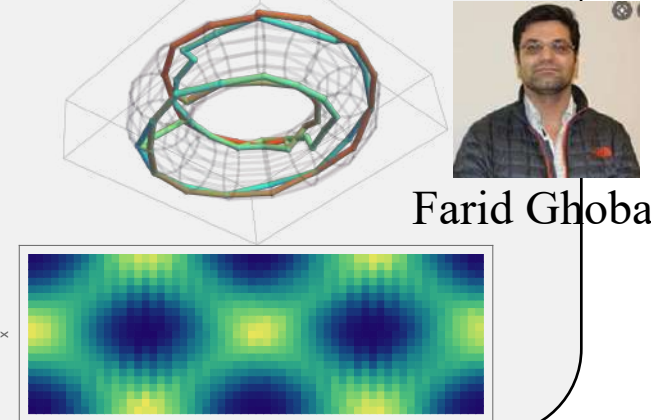


Process tomography

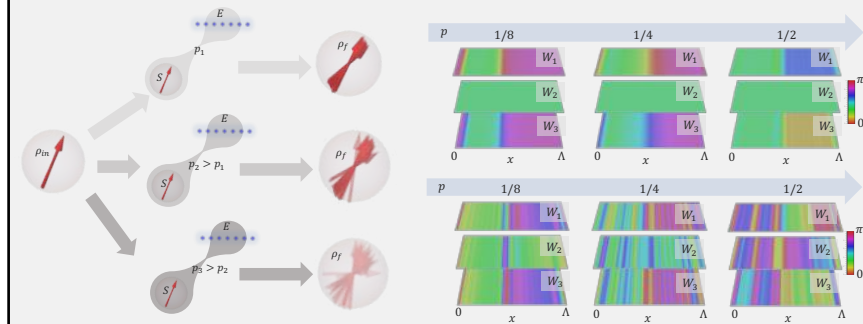


npj Quantum Information 10.1 (2024): 49.

Cyclic quantum walks

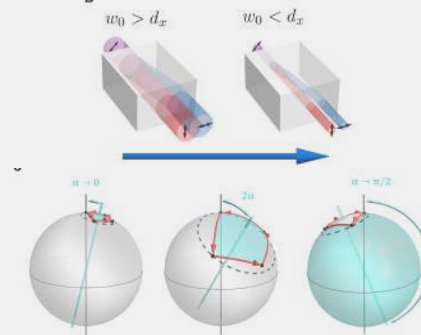


Open systems



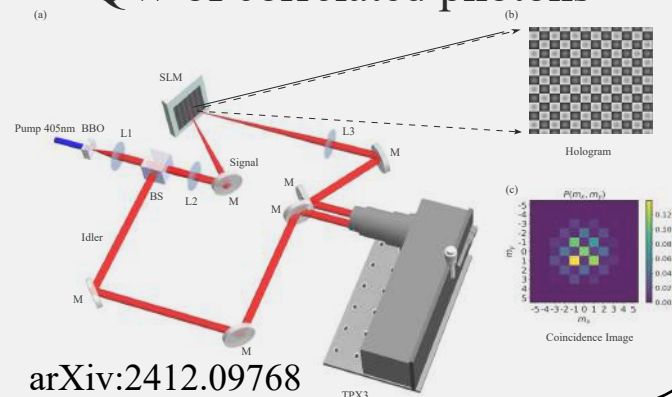
arXiv:2412.04701

Topology and geometric phases



Science advances, 9(47), eadg6810

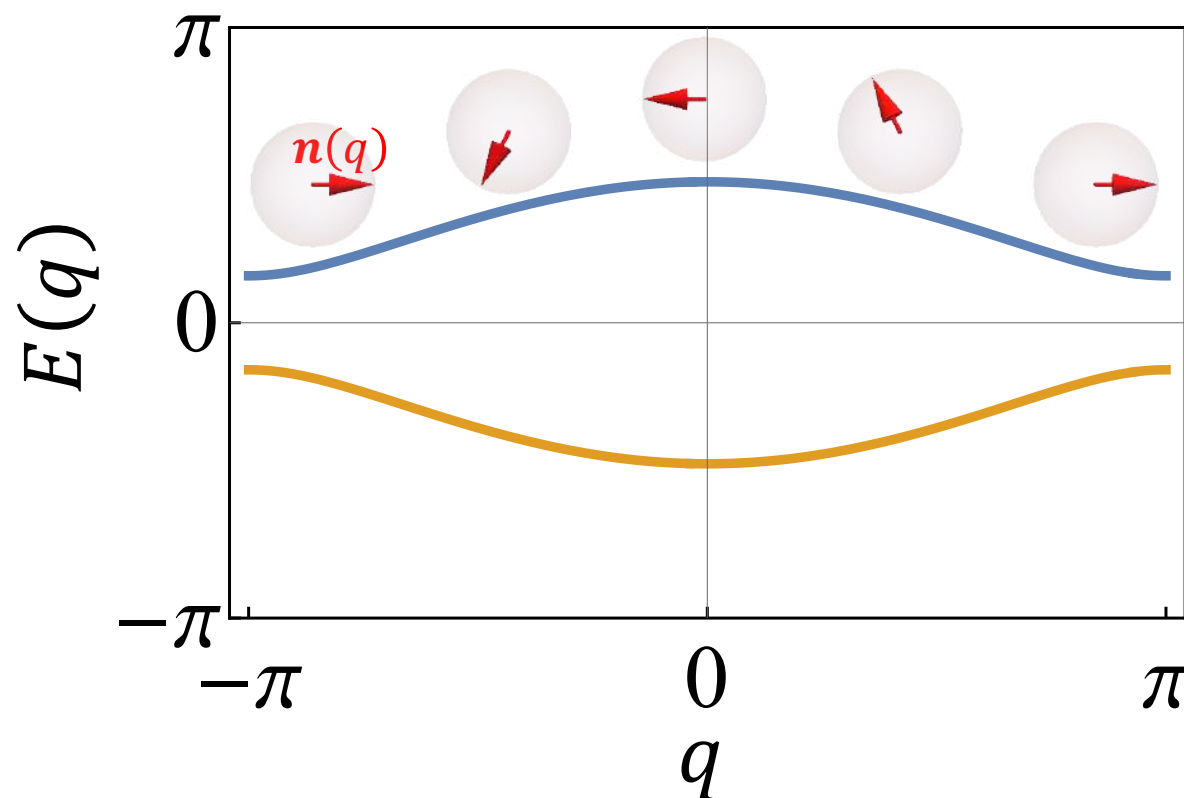
QW of correlated photons



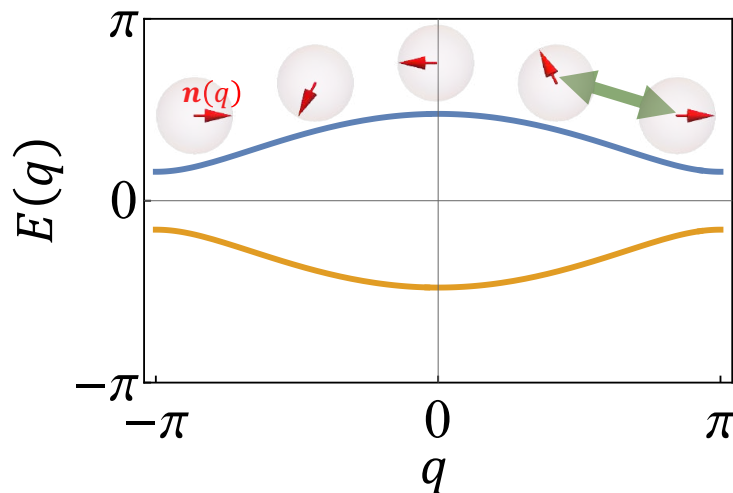
arXiv:2412.09768

Geometry of $\mathcal{U}(q)$

$$\mathcal{U}^t(q) = \exp(-i t E(q) \mathbf{n}(q) \cdot \boldsymbol{\sigma})$$



Geometry of $\mathcal{U}(q)$: quantum metric



$distance(|\mathbf{n}(q_1)\rangle, |\mathbf{n}(q_2)\rangle) = ?$

Quantum Geometric Tensor (Fubini Study metric)

$$\eta_{ij}(q) = \langle \partial_i \mathbf{n} | \partial_j \mathbf{n} \rangle - \langle \mathbf{n} | \partial_i \mathbf{n} \rangle \langle \mathbf{n} | \partial_j \mathbf{n} \rangle$$

$$d\ell^2 = \text{Re}[\eta_{ij}(q)] dq_i dq_j$$

PHYSICAL REVIEW LETTERS 131, 240001 (2023)

Essay: Where Can Quantum Geometry Lead Us?

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(Received 17 November 2023; published 11 December 2023)

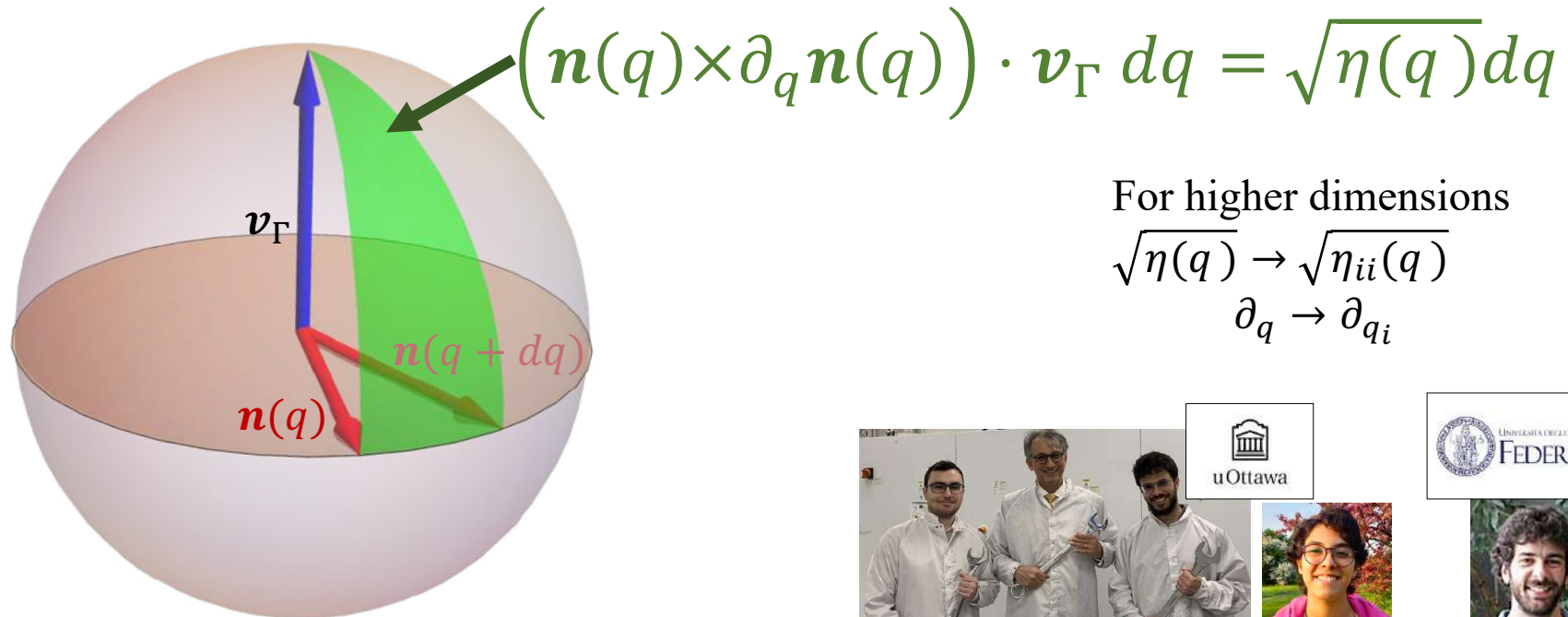
Kang, M., Kim, S., Qian, Y. *et al.* Measurements of the quantum geometric tensor in solids. *Nat. Phys.* (2024). <https://doi.org/10.1038/s41567-024-02678-8>



Provost, J. P., and G. Vallee. "Riemannian structure on manifolds of quantum states." *Communications in Mathematical Physics* 76 (1980): 289-301.

Quantum metric under chiral symmetry

chiral (or sublattice)symmetry: $\mathbf{n}(q) \perp \mathbf{v}_\Gamma$ for each q



For higher dimensions

$$\sqrt{\eta(q)} \rightarrow \sqrt{\eta_{ii}(q)}$$

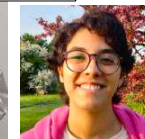
$$\partial_q \rightarrow \partial_{q_i}$$



Francesco Di
Colandrea

Ebrahim
Karimi

Alessio
D'Errico



Nazanin
Dehghan



Filippo
Cardano



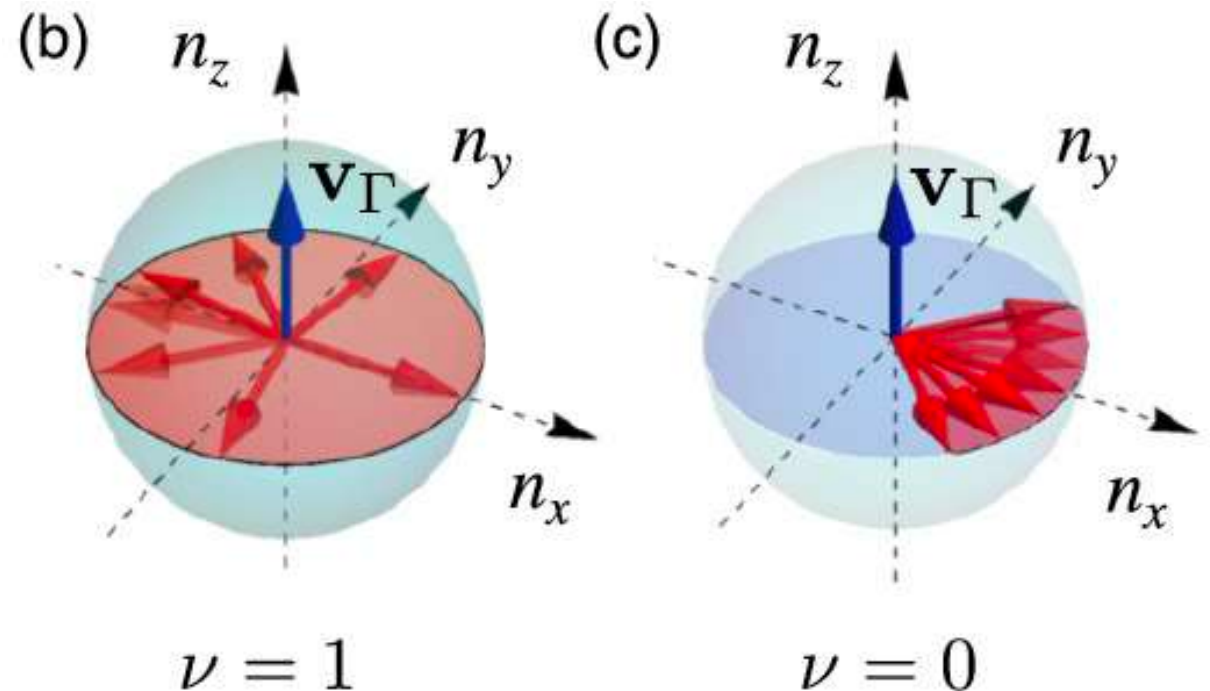
Topology of $\mathcal{U}(q)$ (in 1D)

chiral symmetry: $\mathbf{n}(q) \perp \mathbf{v}_\Gamma$ for each q

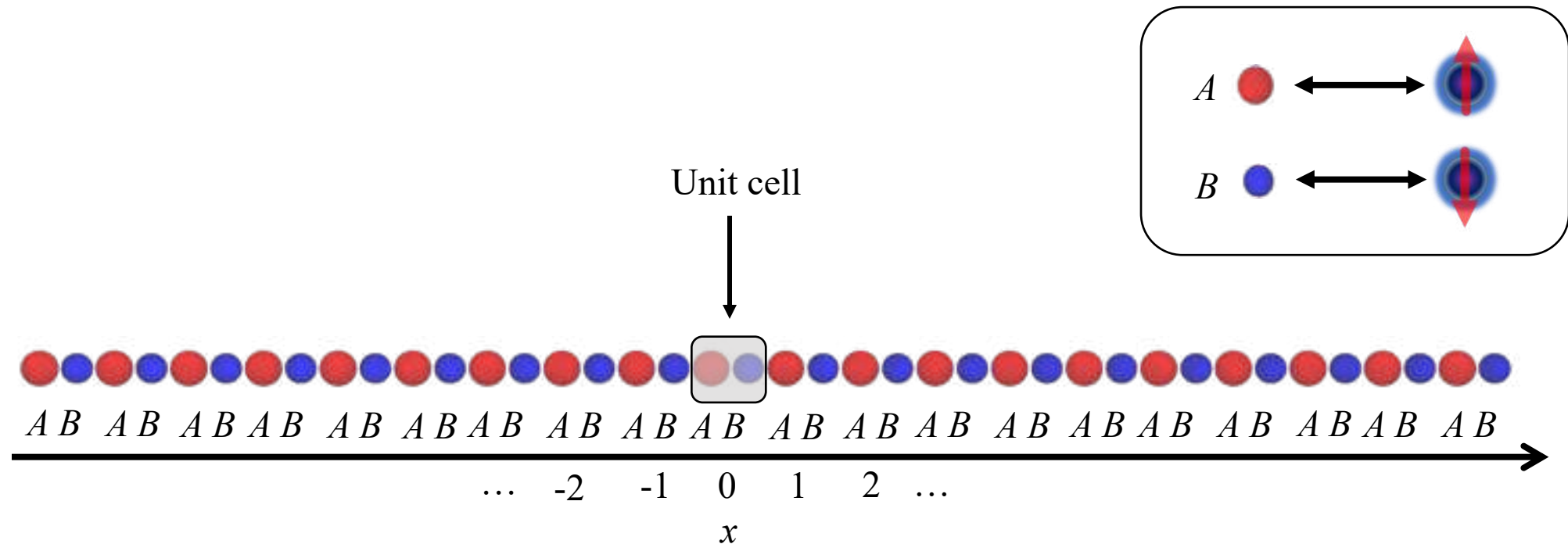
Winding number:

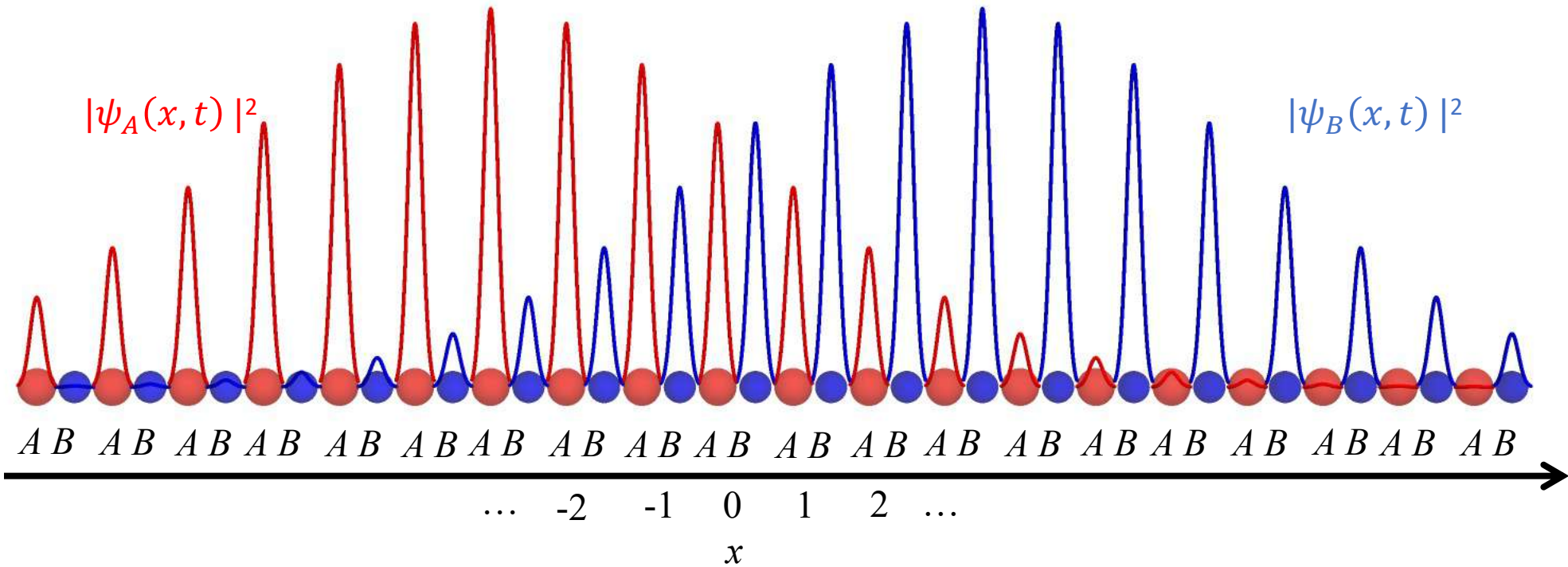
$$\nu = \int_{-\pi}^{\pi} \sqrt{\eta(q)} \frac{dq}{2\pi}$$

$$\sqrt{\eta(q)} = \partial_q \phi$$



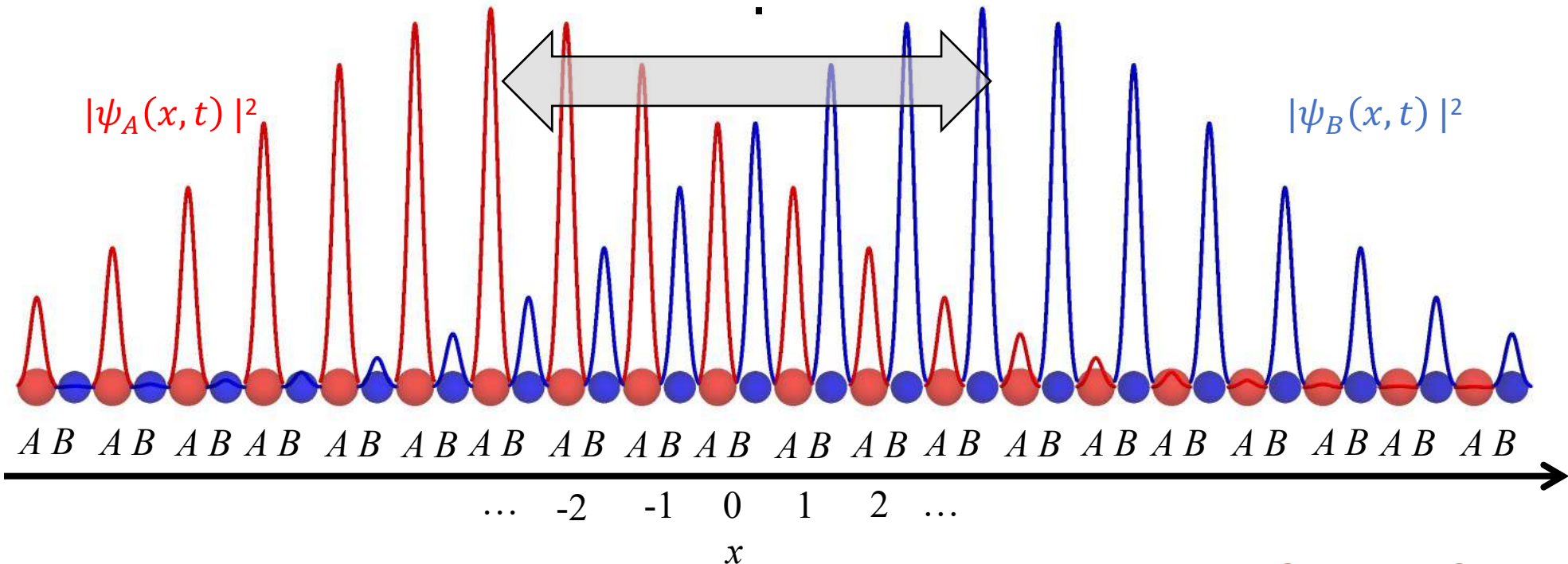
Topology and Geometry in Chiral symmetric Systems





Mean chiral displacement

$$\mathcal{C}(t) := \langle x \rangle_B - \langle x \rangle_A$$



$$\eta_{ij} = ?, v = ?$$



Mean chiral displacement measures winding numbers

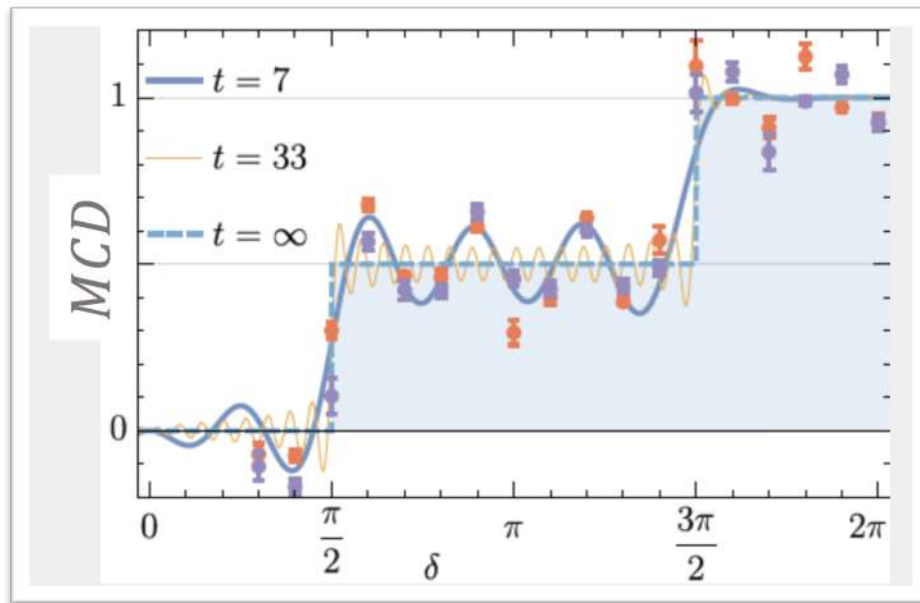
$$\mathcal{C}(t) := \langle \psi(t) | \Gamma x | \psi(t) \rangle$$

$$\Gamma := |B\rangle\langle B| - |A\rangle\langle A|.$$

$$\mathcal{C}(t) \stackrel{t \gg 0}{\sim} \nu/2$$

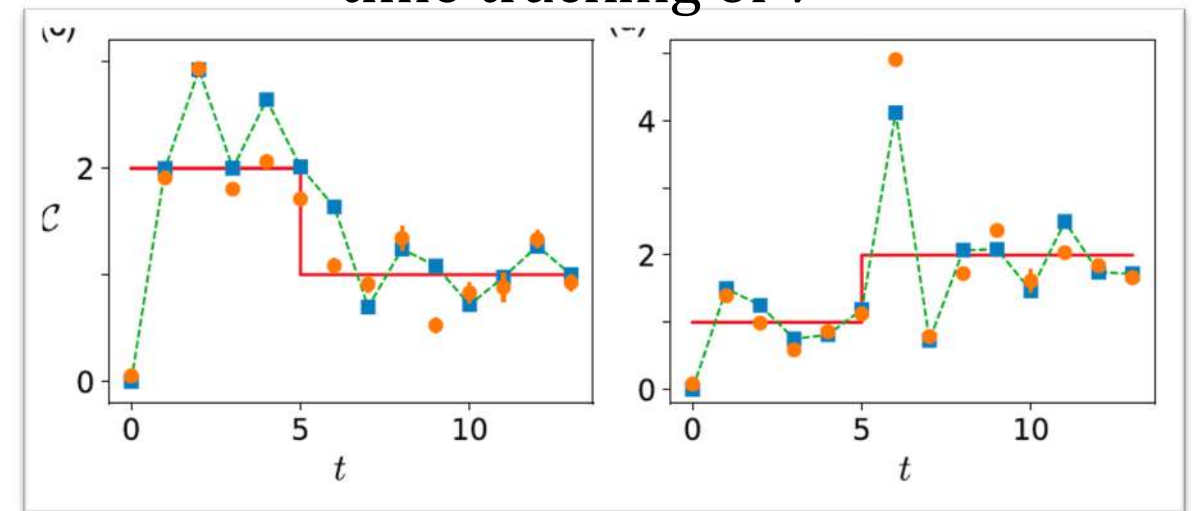
$$|\psi(t=0)\rangle = |x=0\rangle \otimes |c_0\rangle$$

Bulk measurement of ν



Nature communications 8.1 (2017): 15516.

time tracking of ν



Physical Review Research 2.2 (2020): 023119

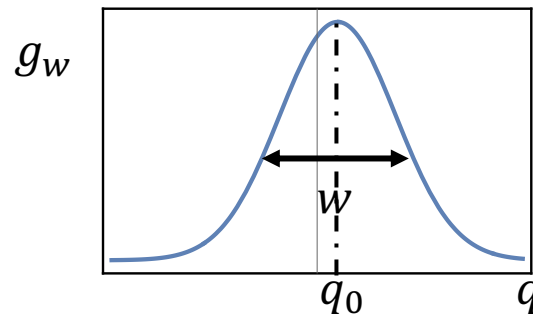


Mean chiral displacement

$$\mathcal{C}(t) := \langle \psi(t) | \Gamma x | \psi(t) \rangle$$

$$\Gamma := |B\rangle\langle B| - |A\rangle\langle A|.$$

$$|\psi(t=0)\rangle = \int g_w(q - q_0) |q\rangle dq \otimes |c_0\rangle$$



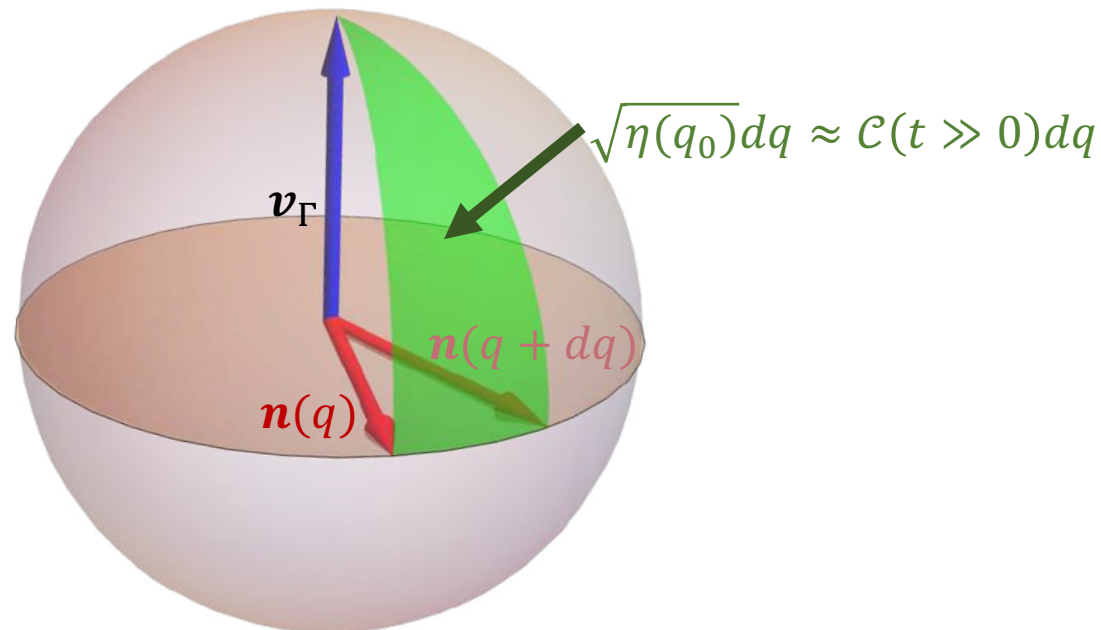
$$\mathcal{C}(t) = \int g_w^2(q - q_0) \sqrt{\eta(q)} \frac{dq}{2} + \frac{f(t)}{\sqrt{t}}$$



Mean chiral displacement of initially delocalized states

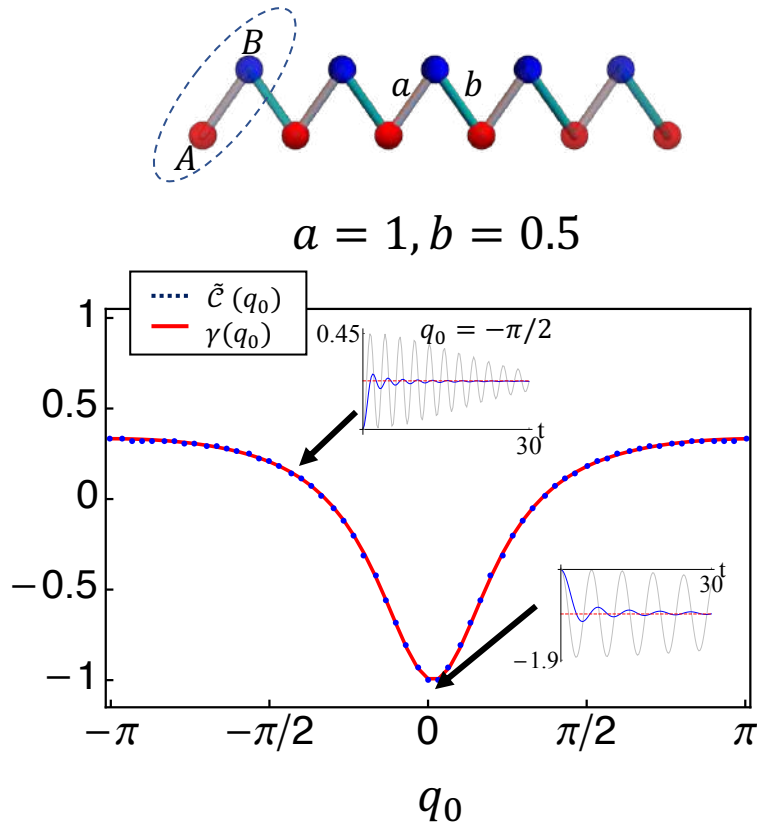
$$\mathcal{C}(t)^{t \gg 0} \approx \int g_w^2(q - q_0) \sqrt{\eta(q)} dq / 2$$

$$w \approx 0 \rightarrow \mathcal{C}(t \gg 0) \sim \sqrt{\eta(q_0)}$$

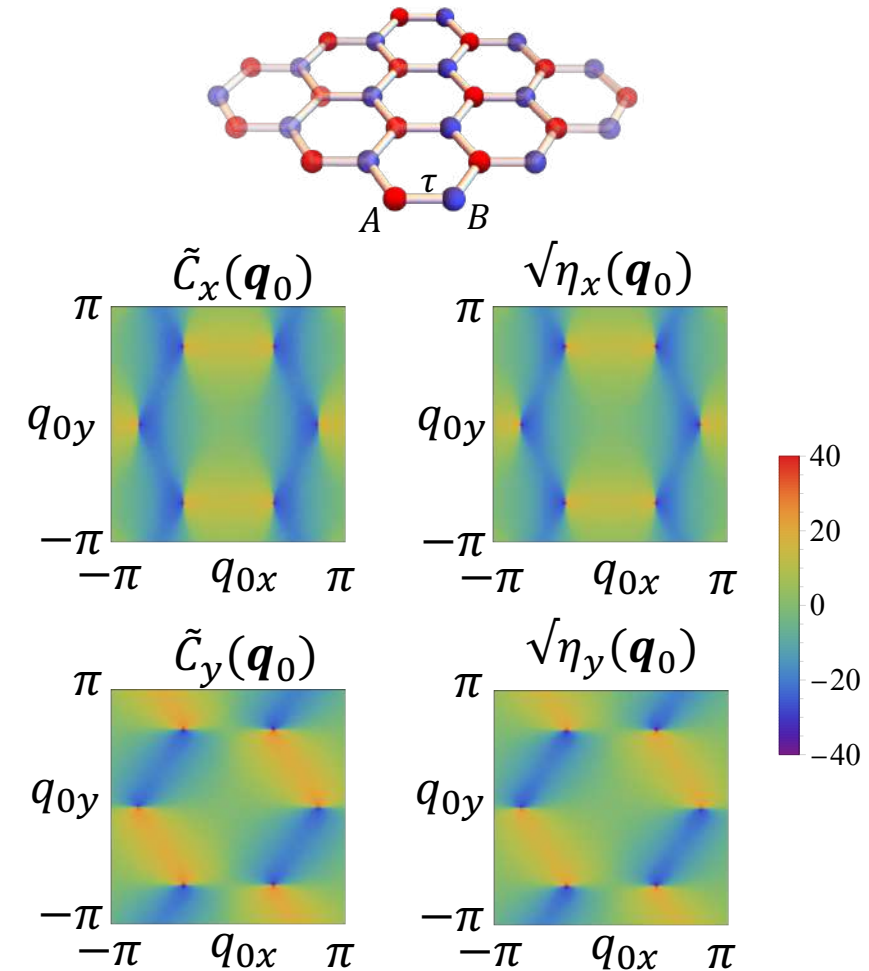


MCD and quantum metric

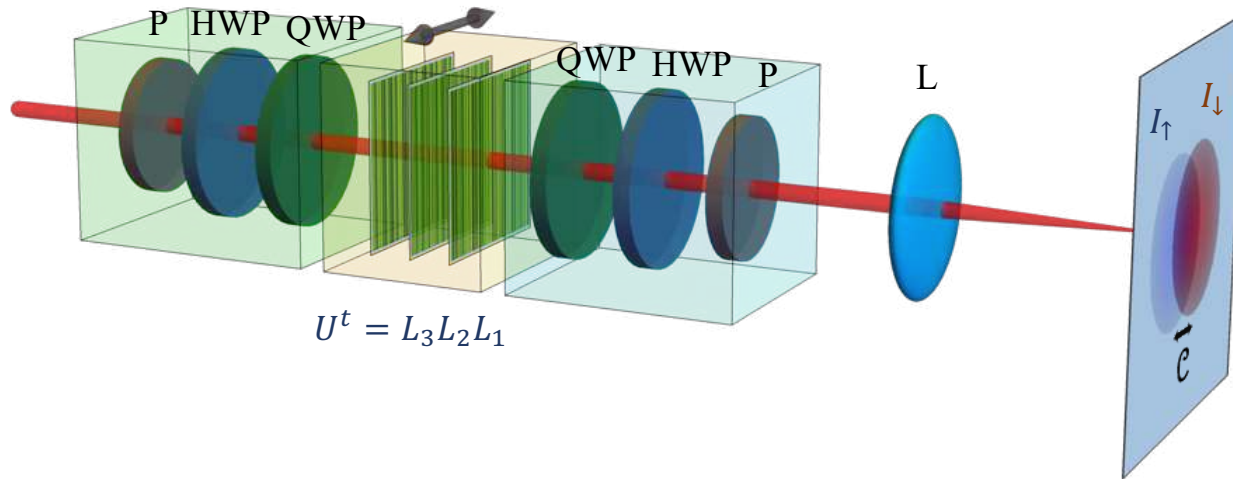
SSH model



Graphene



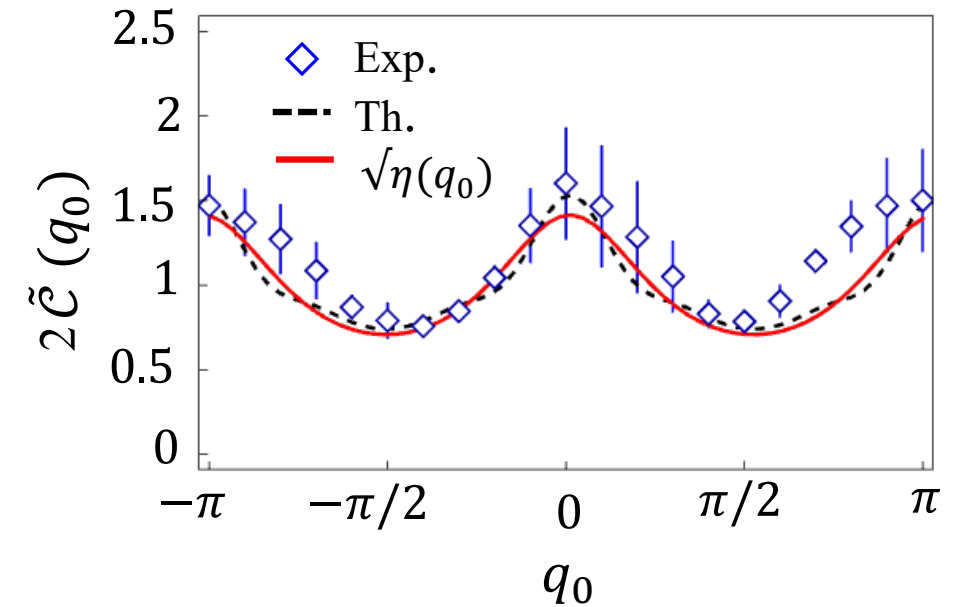
MCD and quantum metric



$$\hat{S} = \sum_x (|x+1\rangle\langle x| \otimes |R\rangle\langle L| + h.c.);$$

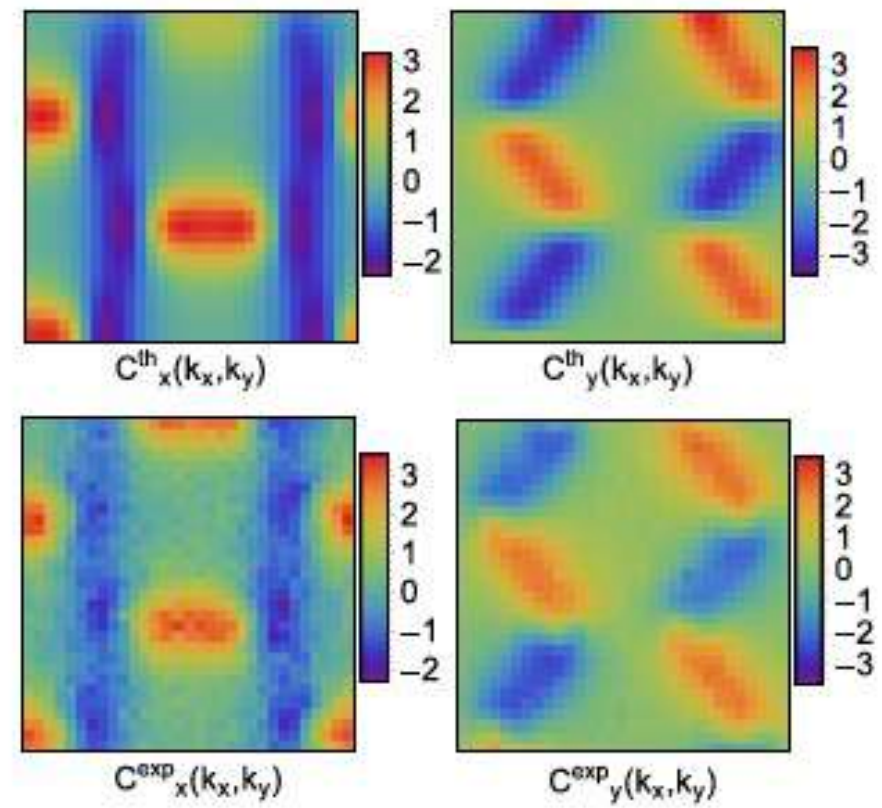
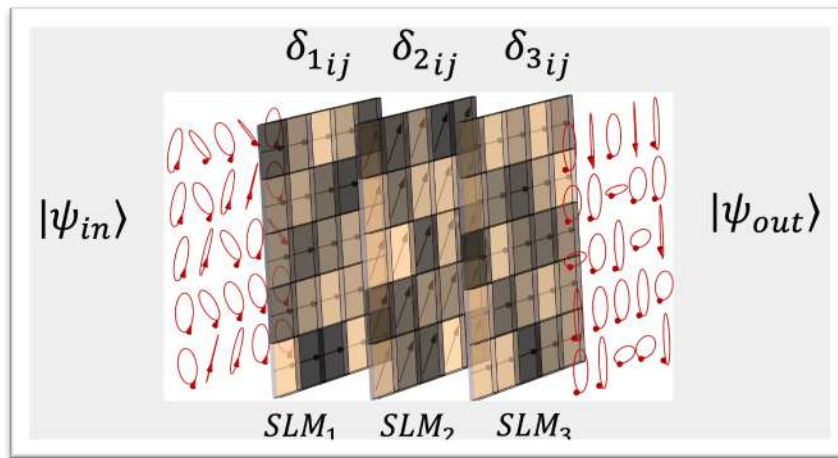
$$\hat{W} = (1 + i \sigma_x) / \sqrt{2}$$

$$\hat{U} = \hat{S} \cdot \hat{W}$$



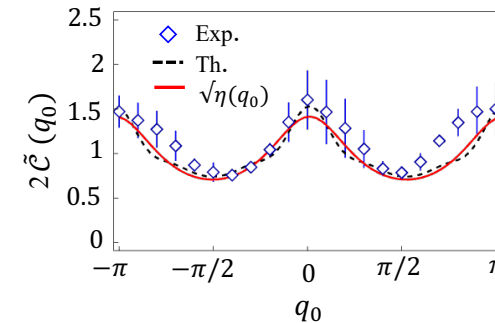
MCD and quantum metric

Graphene Hamiltonian

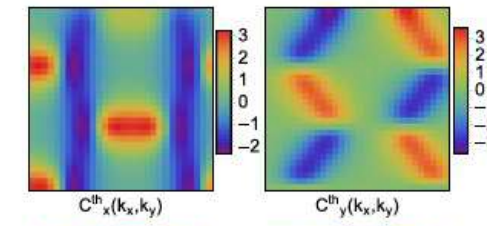


Conclusions and prospects

The quantum metric of chiral symmetric systems can be extracted from the MCD



The distribution of a wavepacket on two sublattices is directly affected by the quantum metric

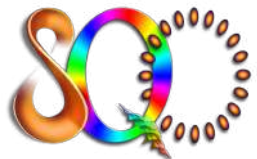


To do: extension of the results to *higher-dimensional internal states*, *non-hermitian dynamics*, and *interacting systems*

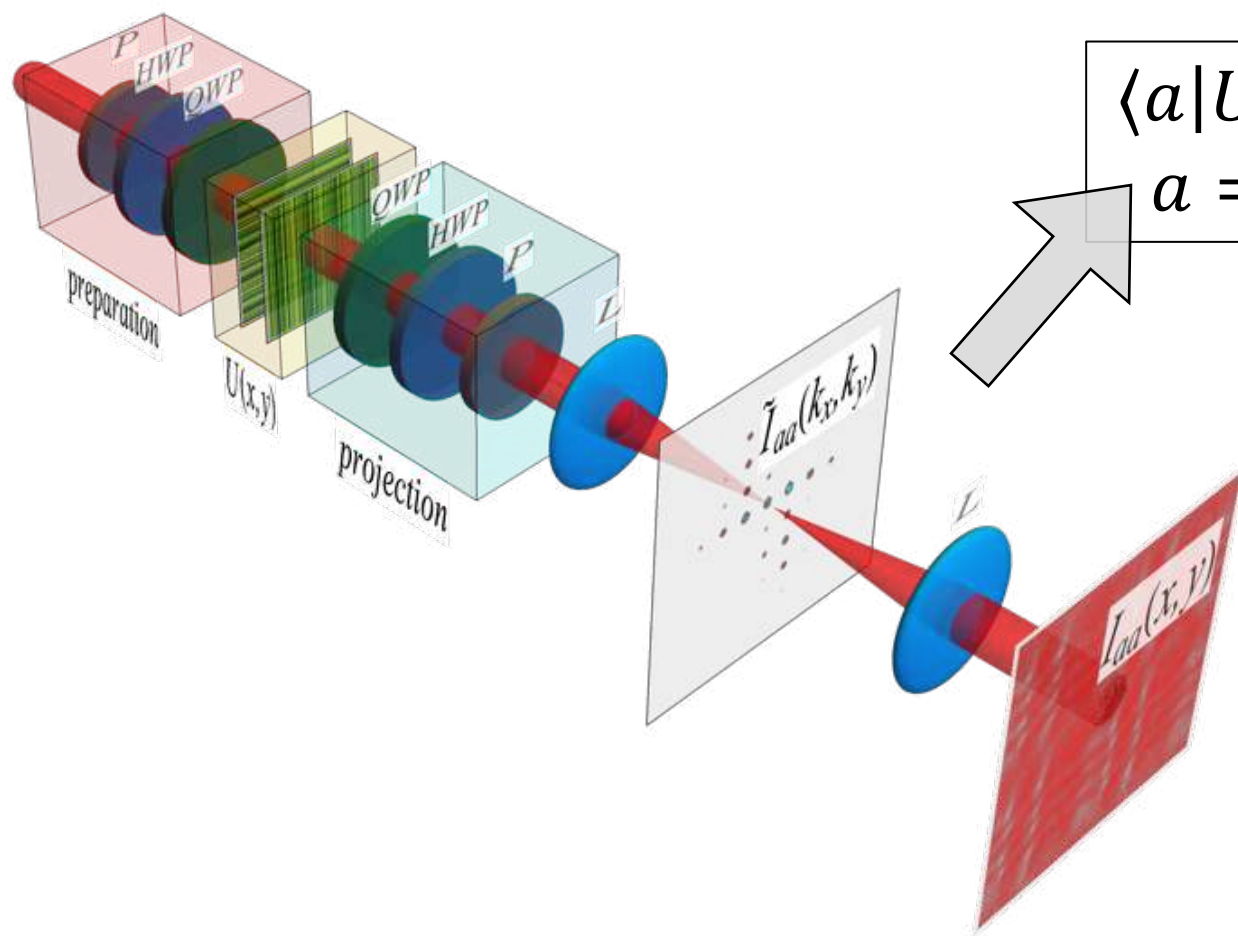


PHD and Postdoc Positions Available

ekarimi@uottawa.ca



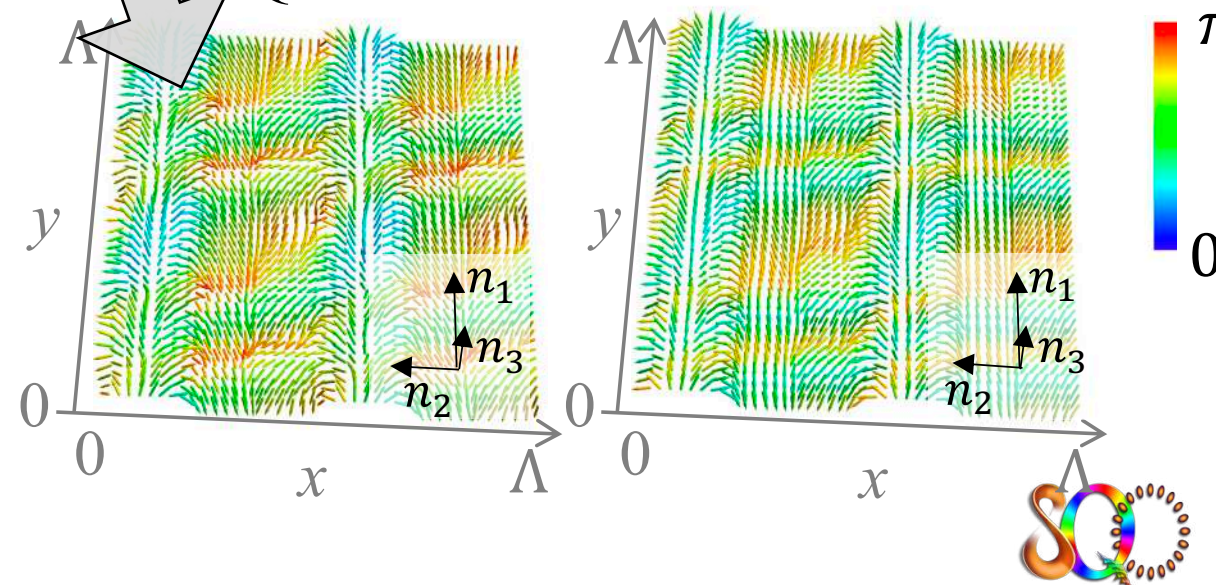
Constructing the full unitary: Fourier Quantum Process Tomography



$$\langle a|U(x, y)|a\rangle$$

$a = H, D, L$

FQPT



ML

A

 $\mathcal{T}$

0