Quantum cellular automata for quantum error correction and density classification[1]

T. L. M. Guedes, D. Winter, M. Müller

QSQW 2025

Link to paper

1. Phys. Rev. Lett. 133.150601 (2024)

Quantum error correction

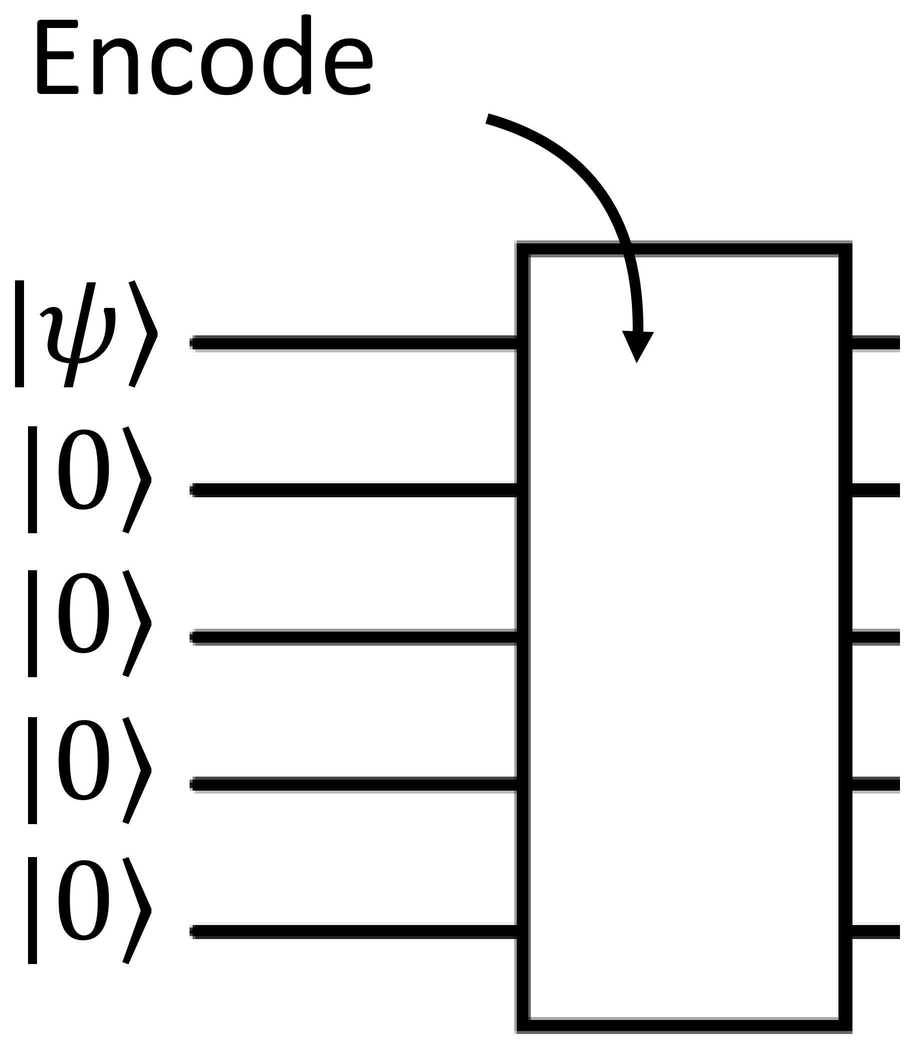
Protect against quantum errors

$|\psi\rangle$ ———

Example

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Quantum error correction



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$$\Big\downarrow \text{Encode}$$

$$|\psi_L\rangle = \alpha|00000\rangle + \beta|11111\rangle$$

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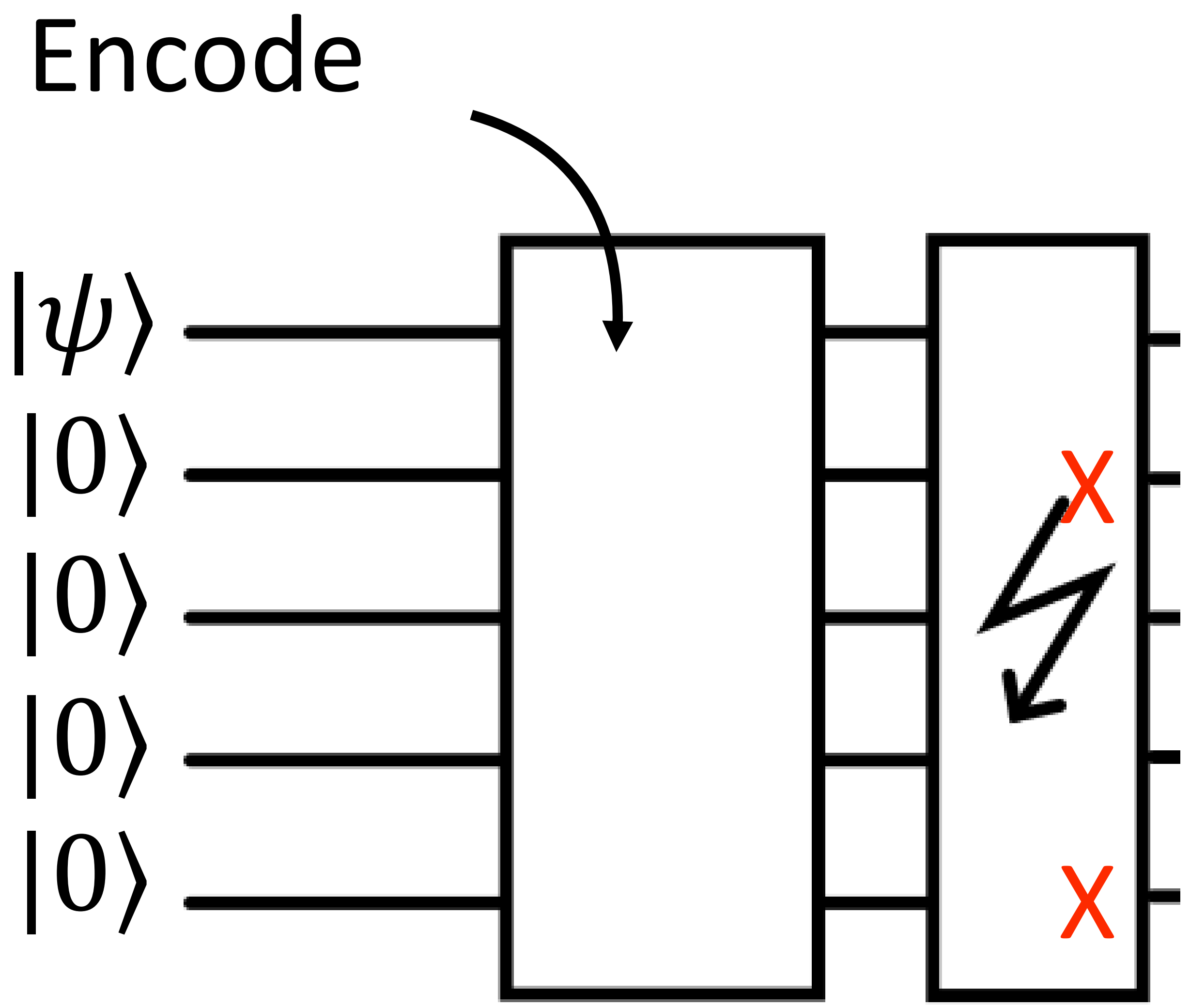

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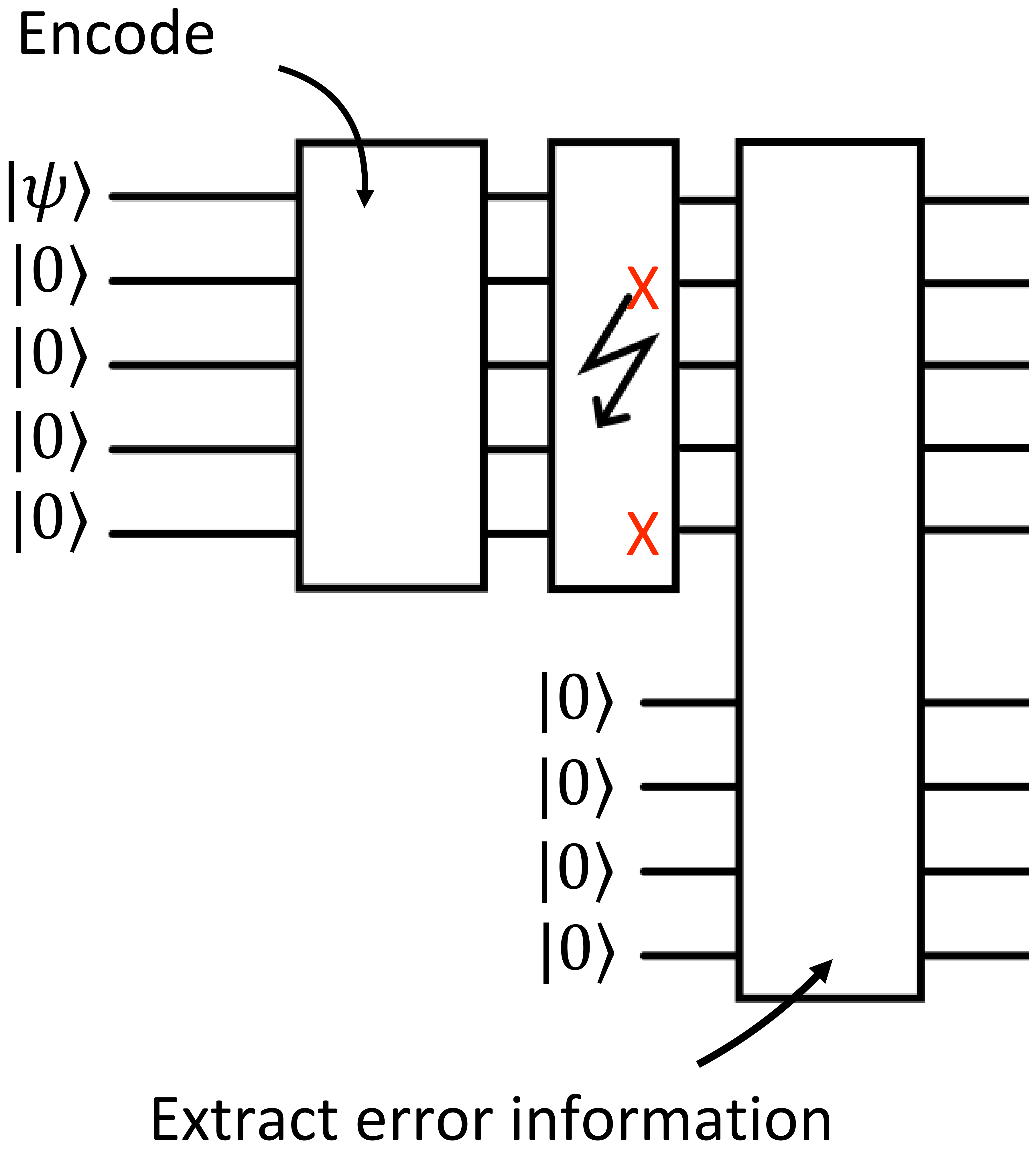

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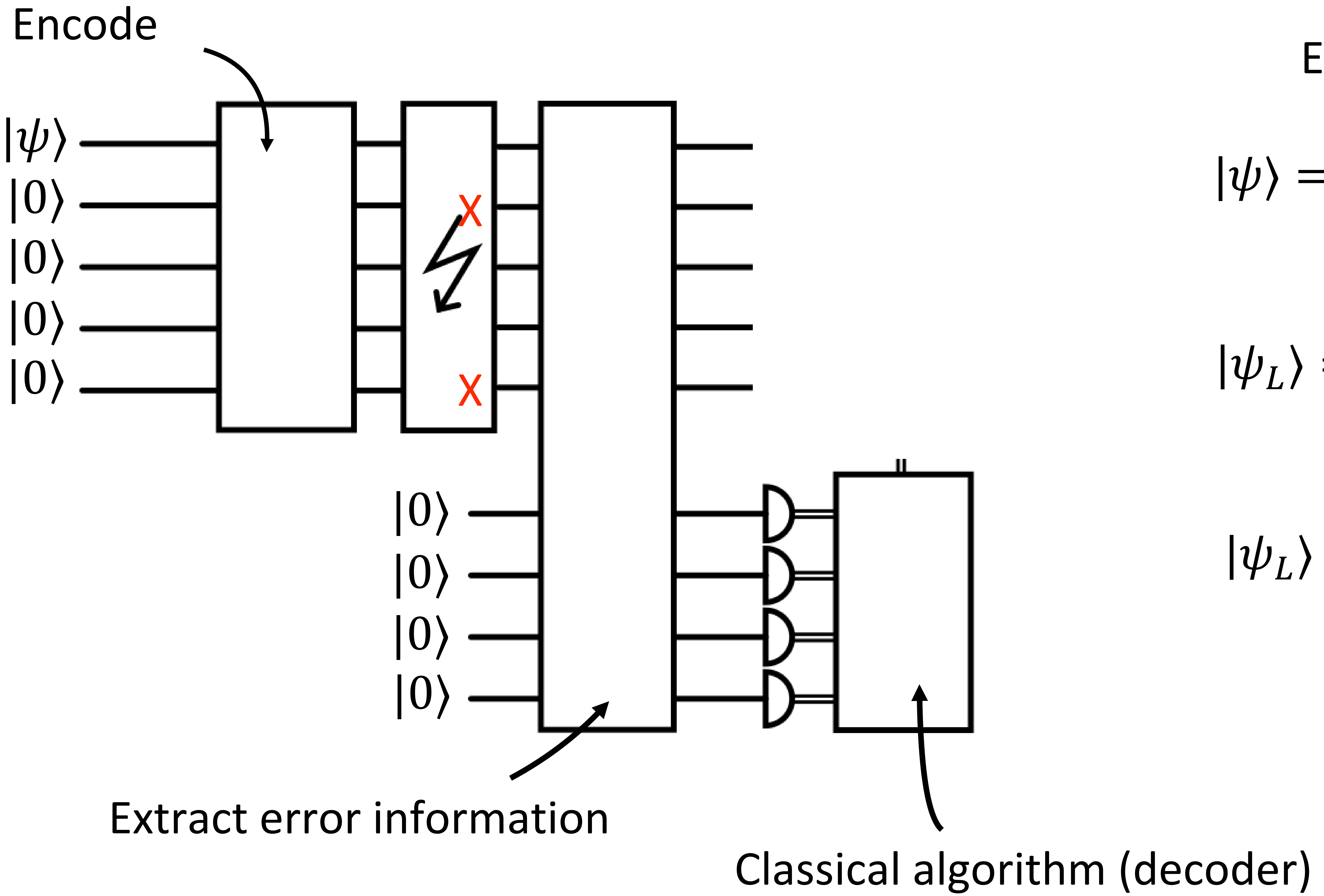

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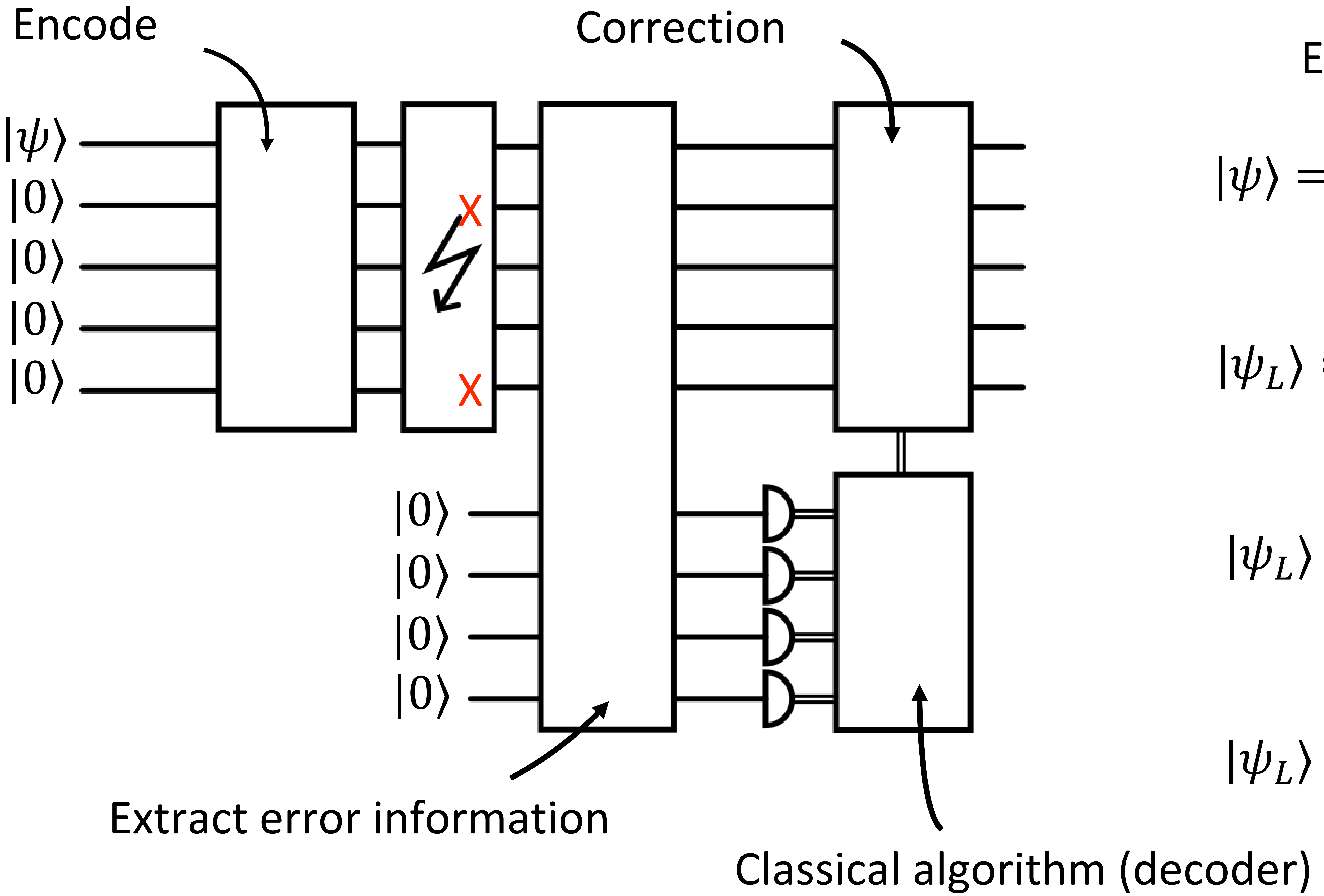

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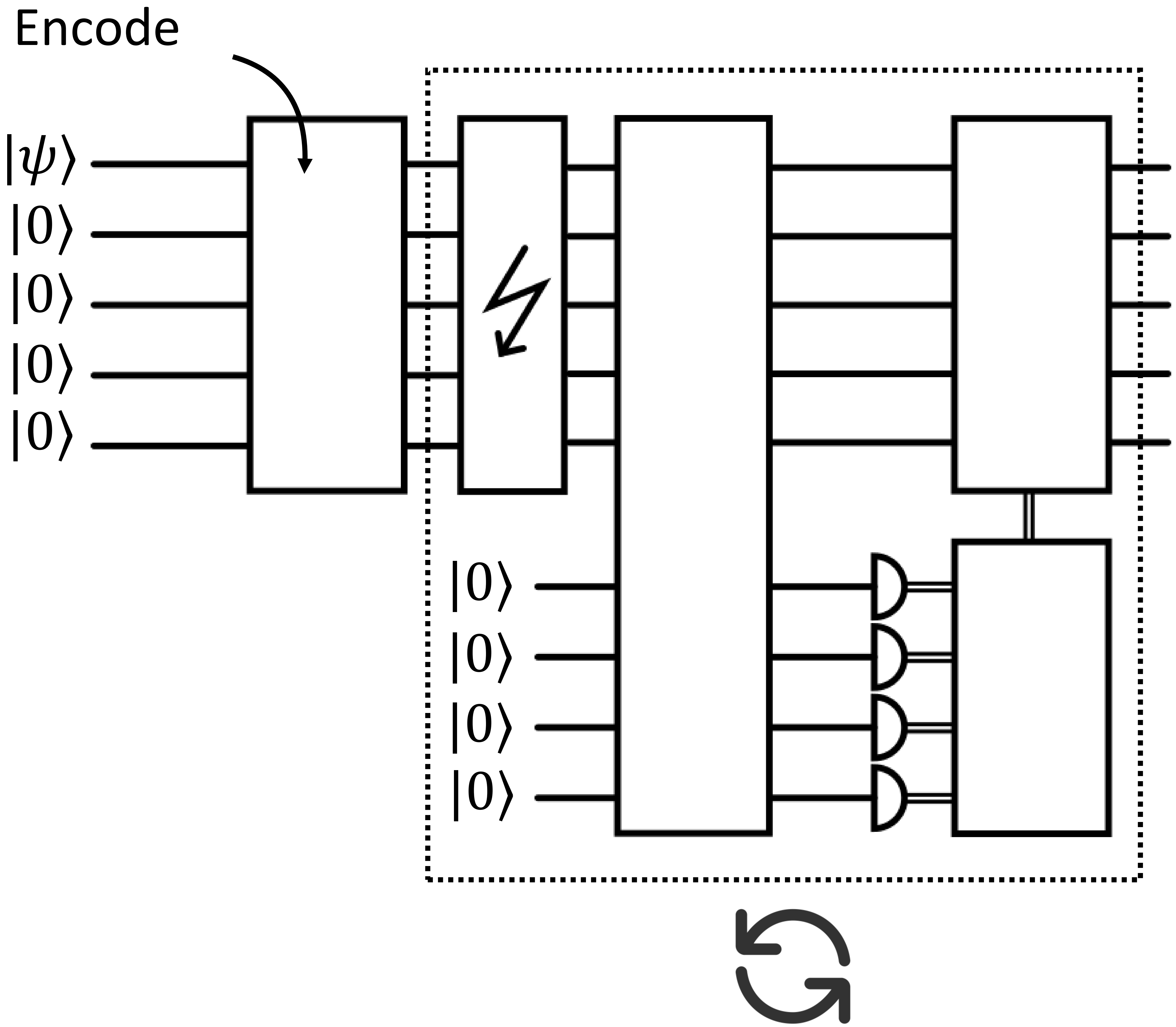

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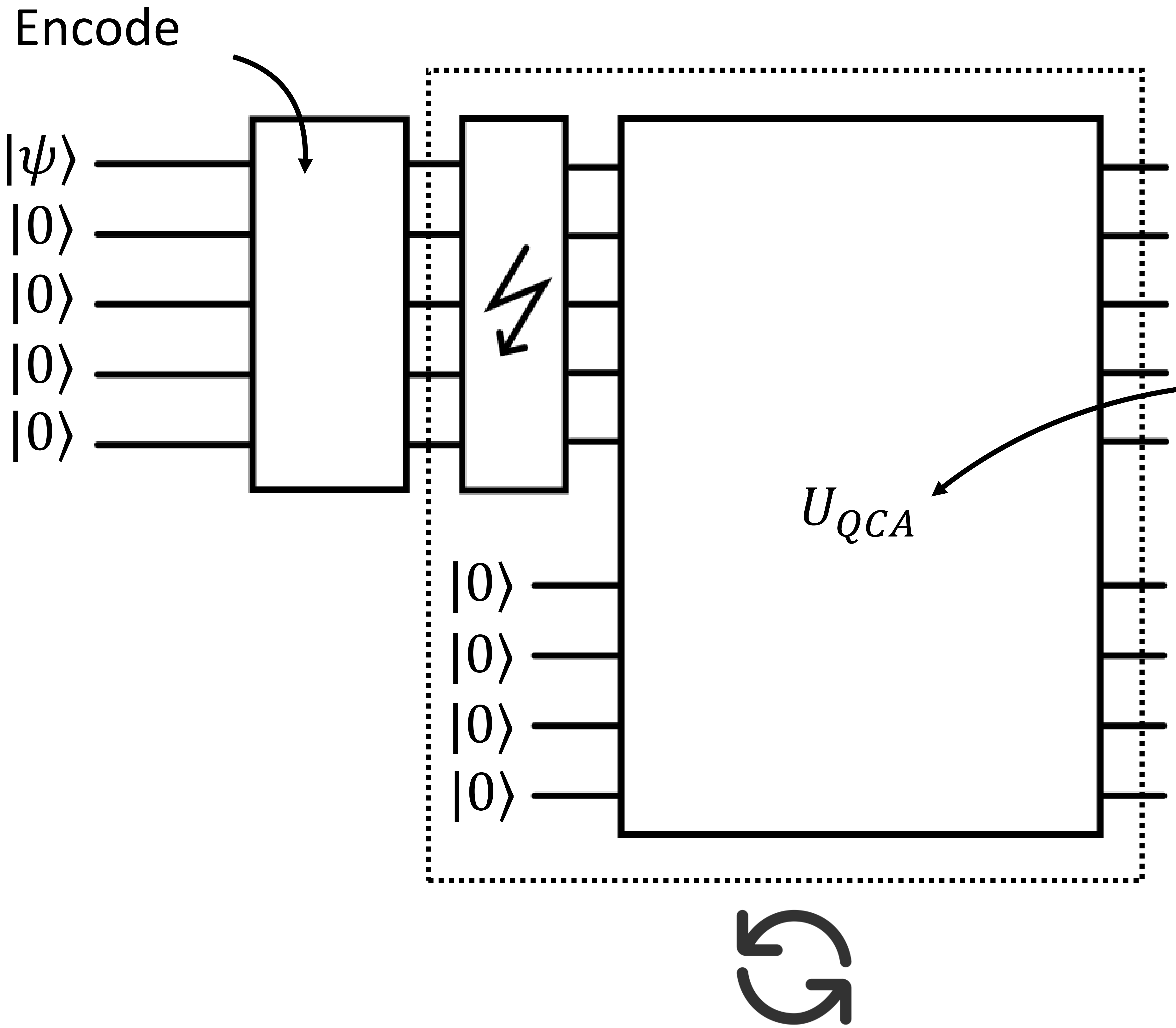
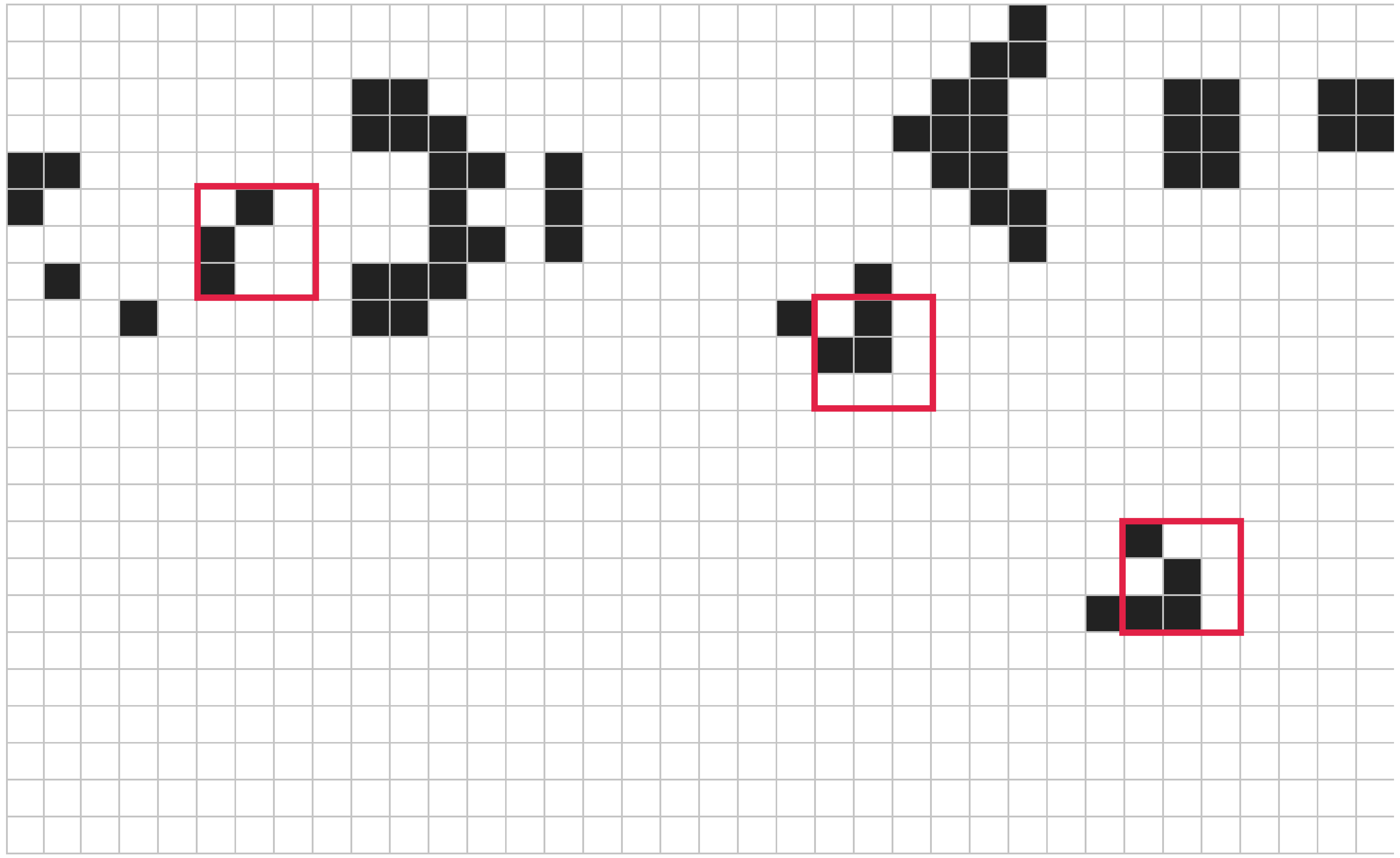


Autonomous, measurement-free quantum cellular automaton dynamics to stabilize logical qubits?

Cellular automata

Conway's game of Life (1970)



Rules

1. Live cell with <2 live neighbors dies (underpopulation)
2. Live cell with 2 or 3 live neighbors stays alive
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4. Dead cell with exactly 3 live neighbors becomes alive (reproduction)

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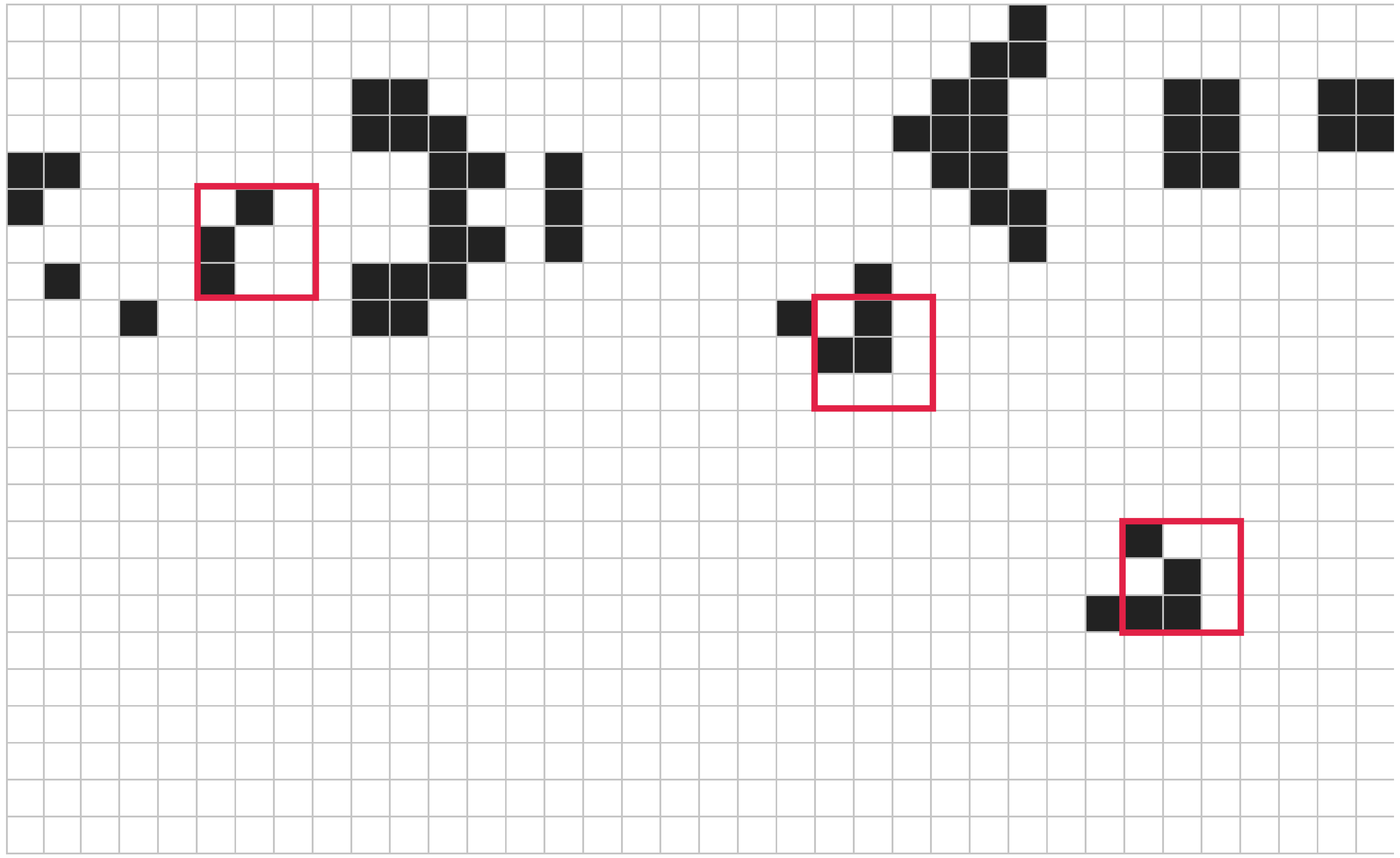

Complex emergent dynamics from
simple local rules

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- Lattice L_d of cells in d dimensions
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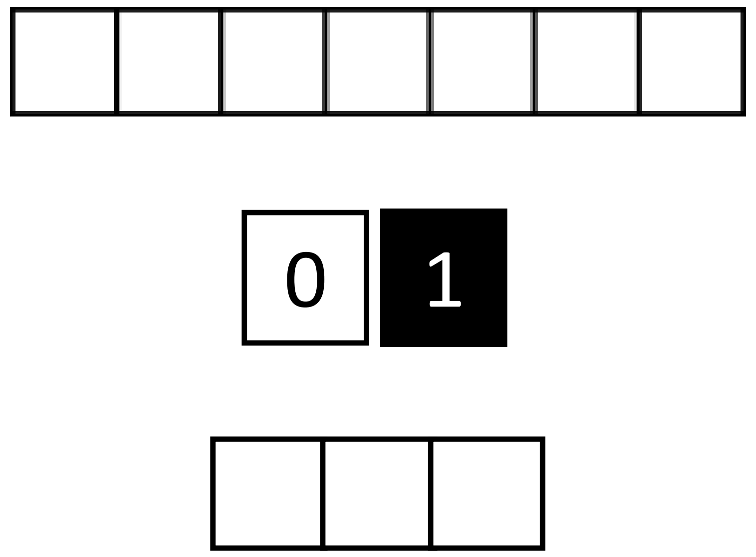
Example: Rule 232



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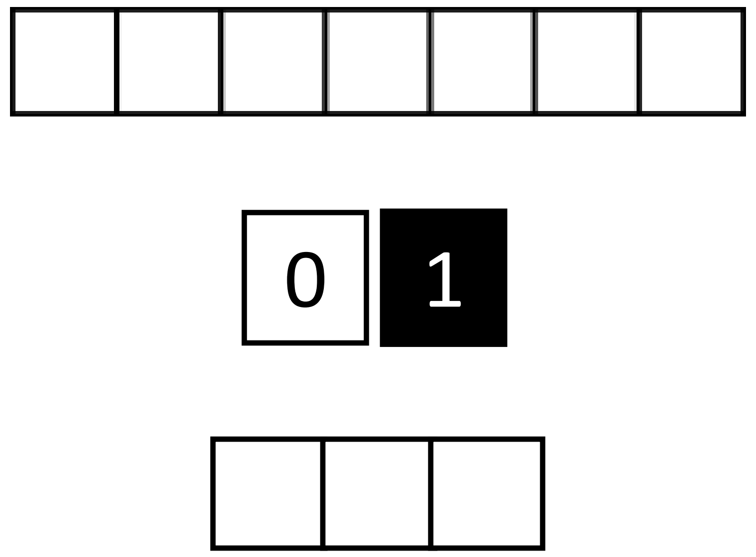

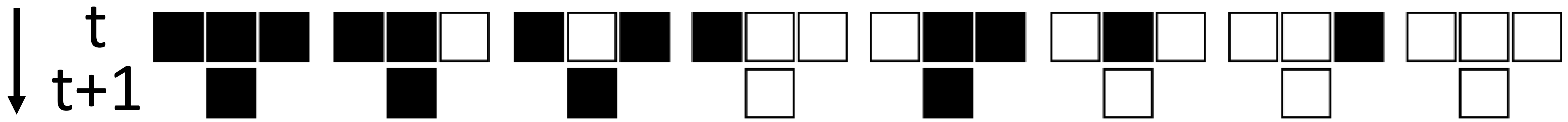

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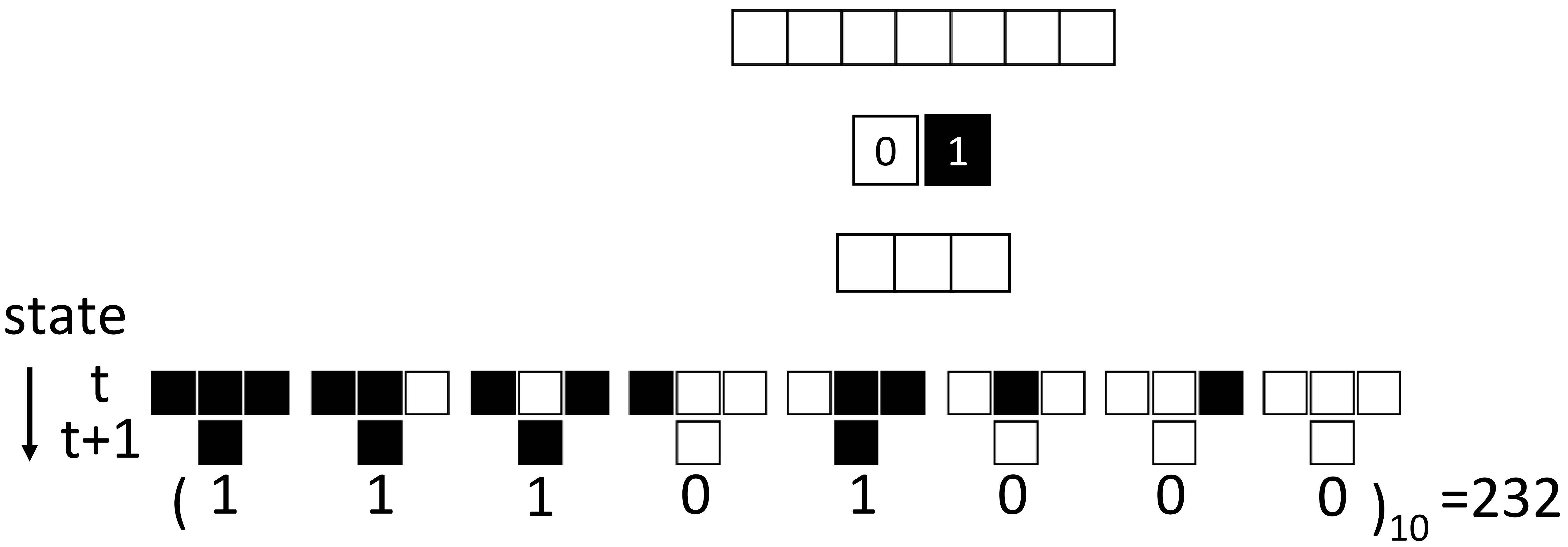

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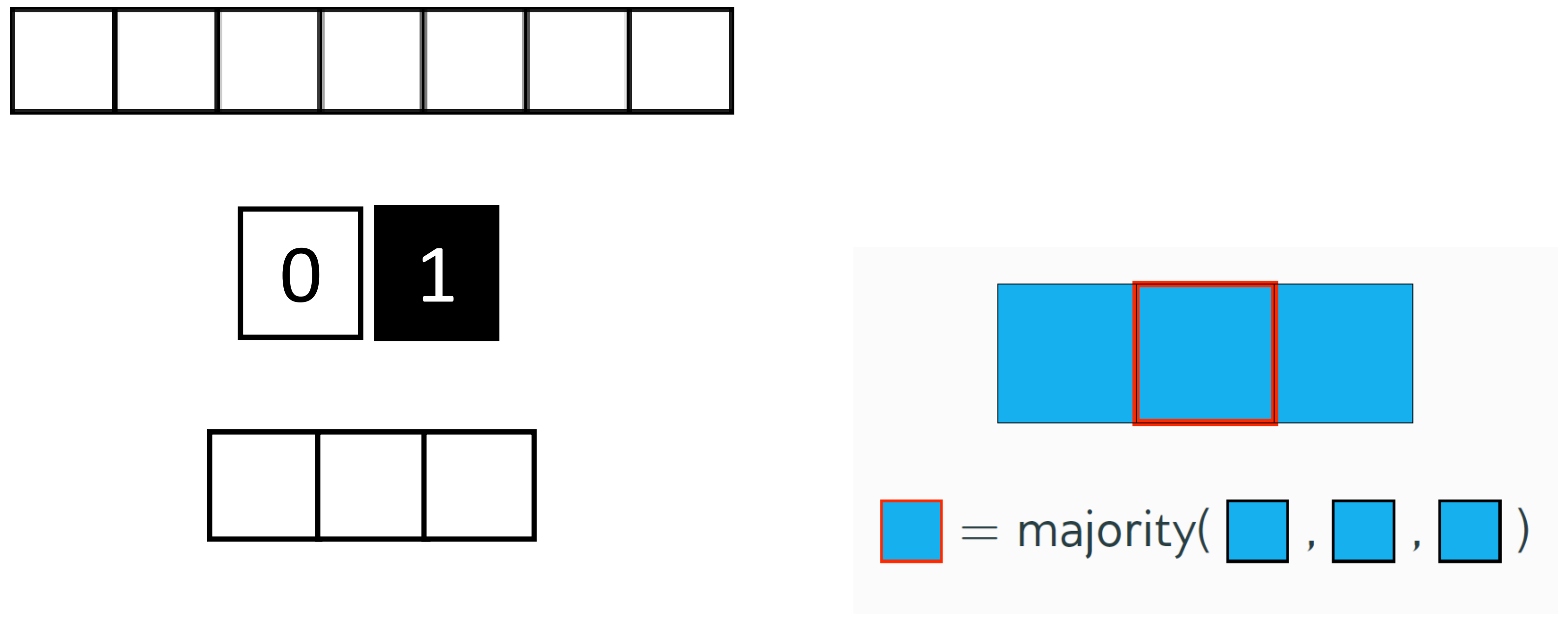

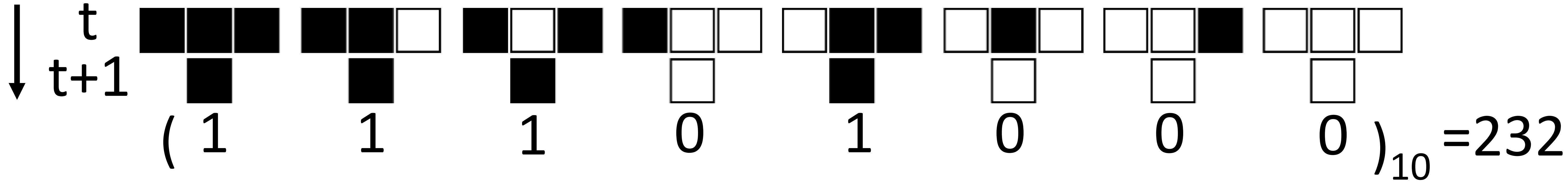

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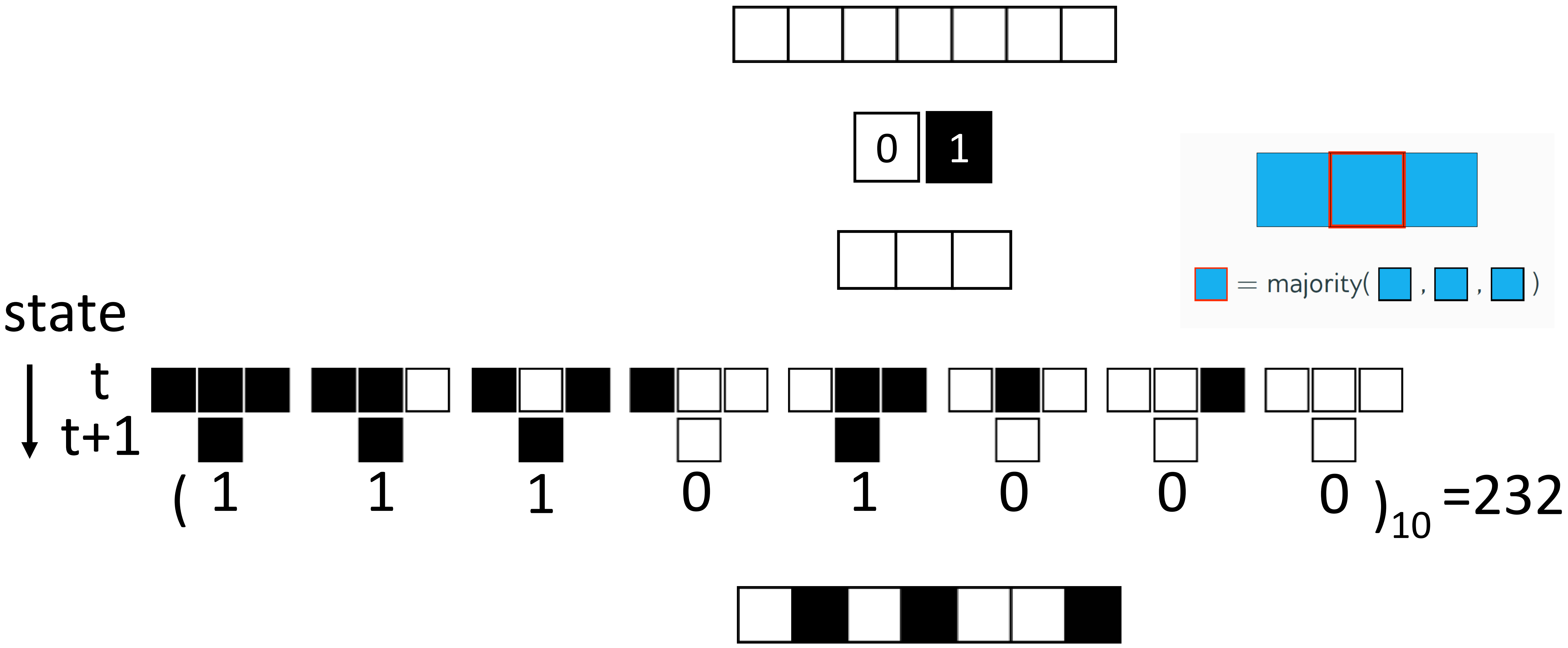

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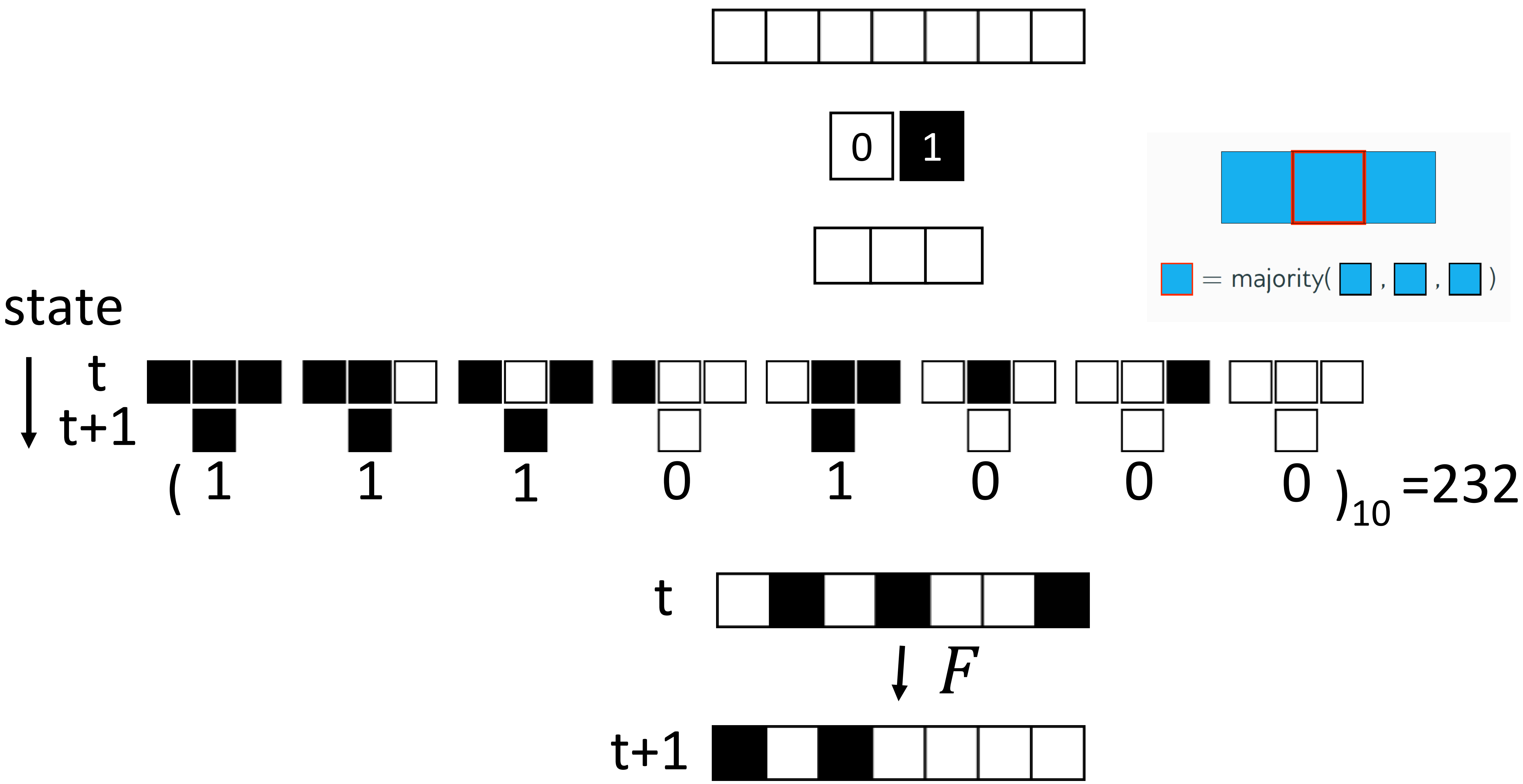
- Global rule $F: Conf \to Conf$
    - Applies local rule synchronously

Example: Rule 232



Cellular automata

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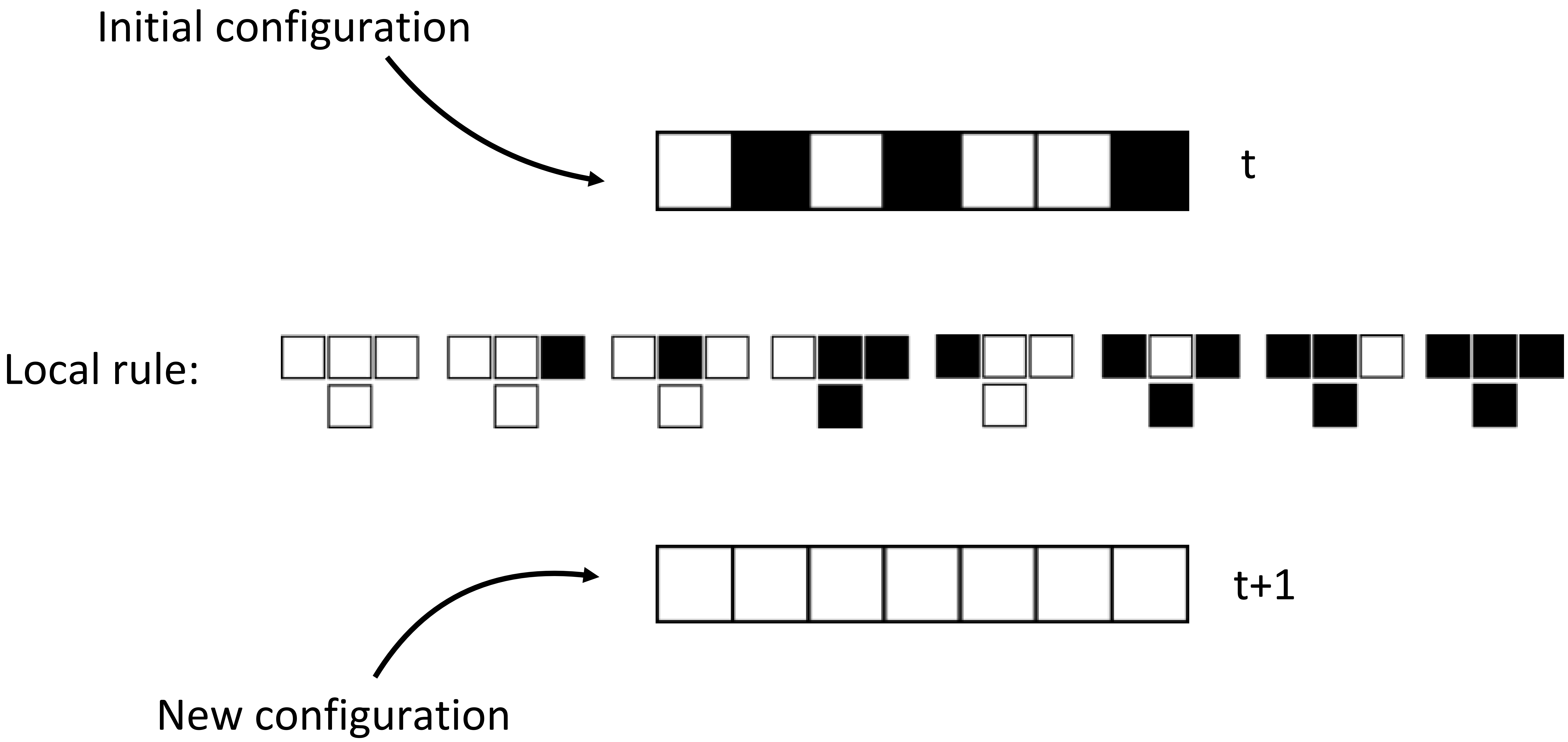

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Density classification

- Want: $|\psi_L\rangle = \alpha|01001\rangle + \beta|10110\rangle \rightarrow |\psi_L\rangle = \alpha|00000\rangle + \beta|11111\rangle$

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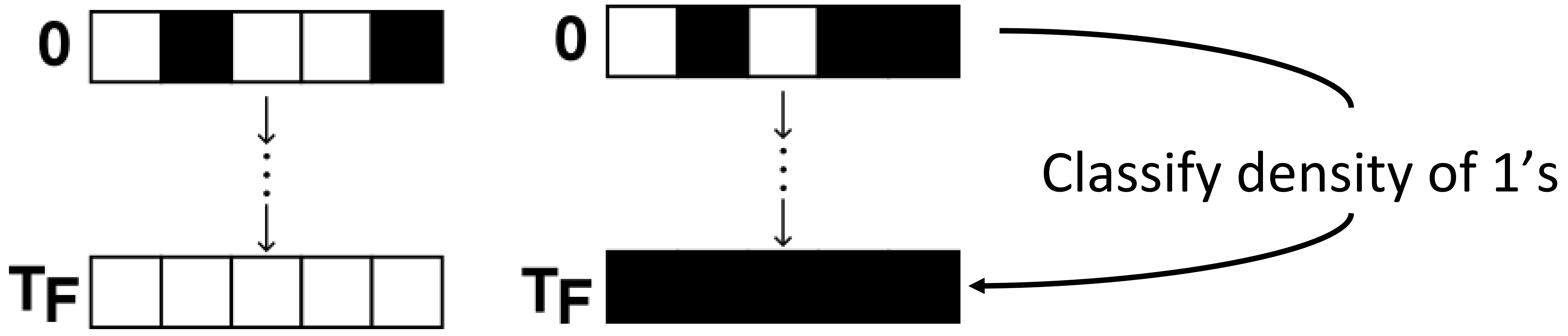
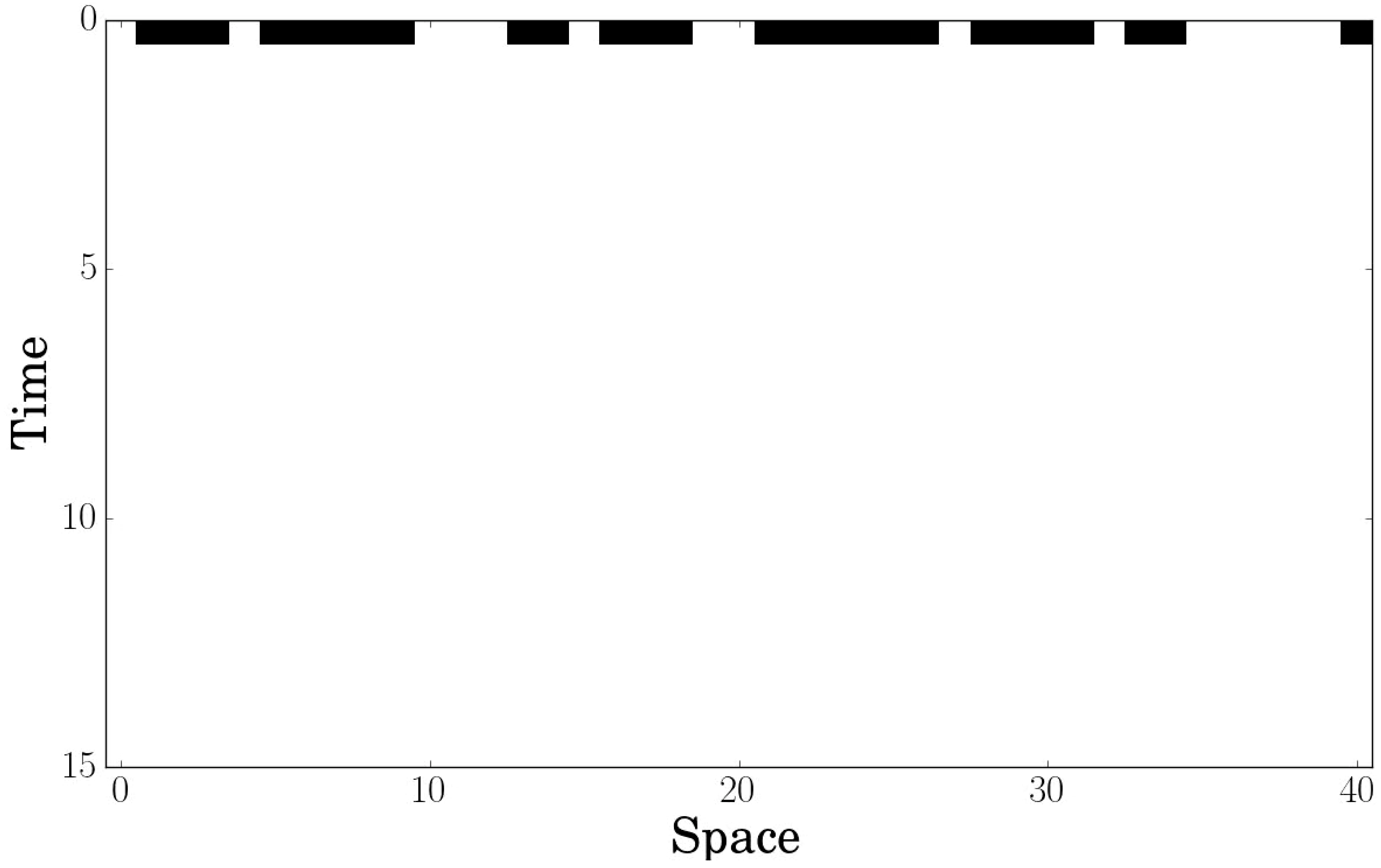
- Classically: Density classification problem
  - Given bit string of $n = i + j$ bits with i zeros and j ones
  - Evolve bit string to all-zeros if $i > j$ and to all-ones if $j > i$



Cellular automata

Is Rule 232 a good density classifier?



Initial configuration

Deterministic evolution

Cellular automata

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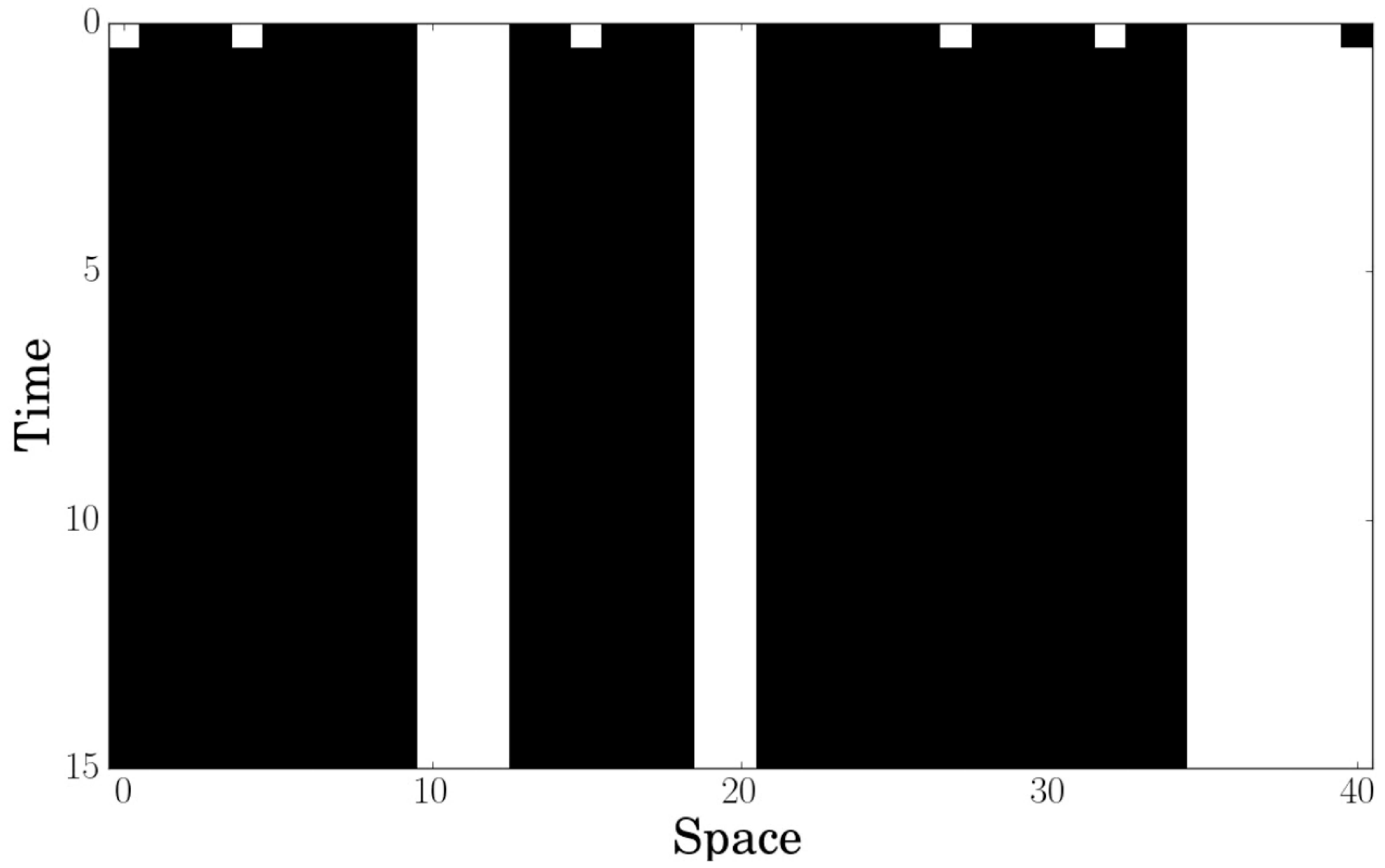


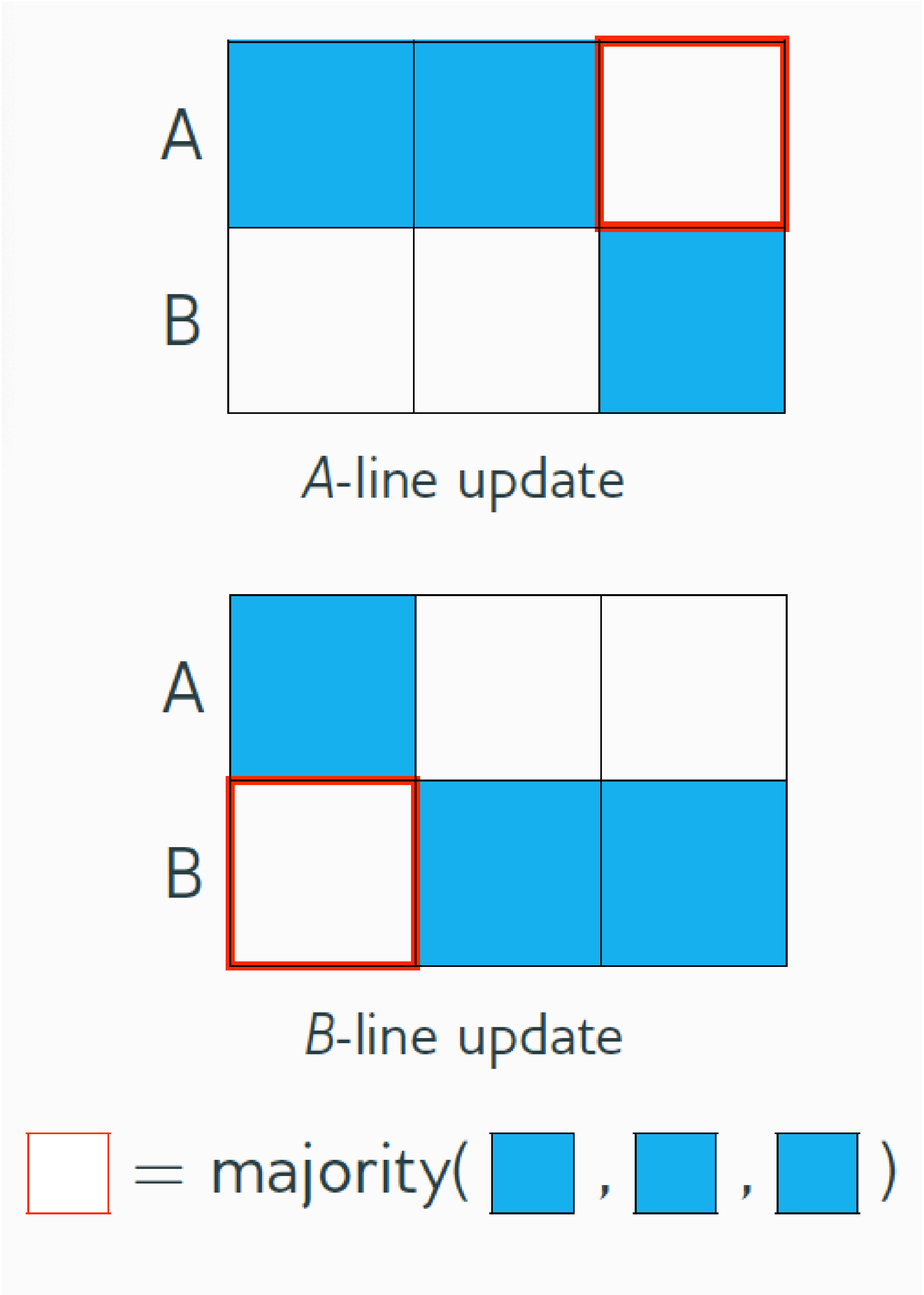
Initial configuration

Deterministic evolution

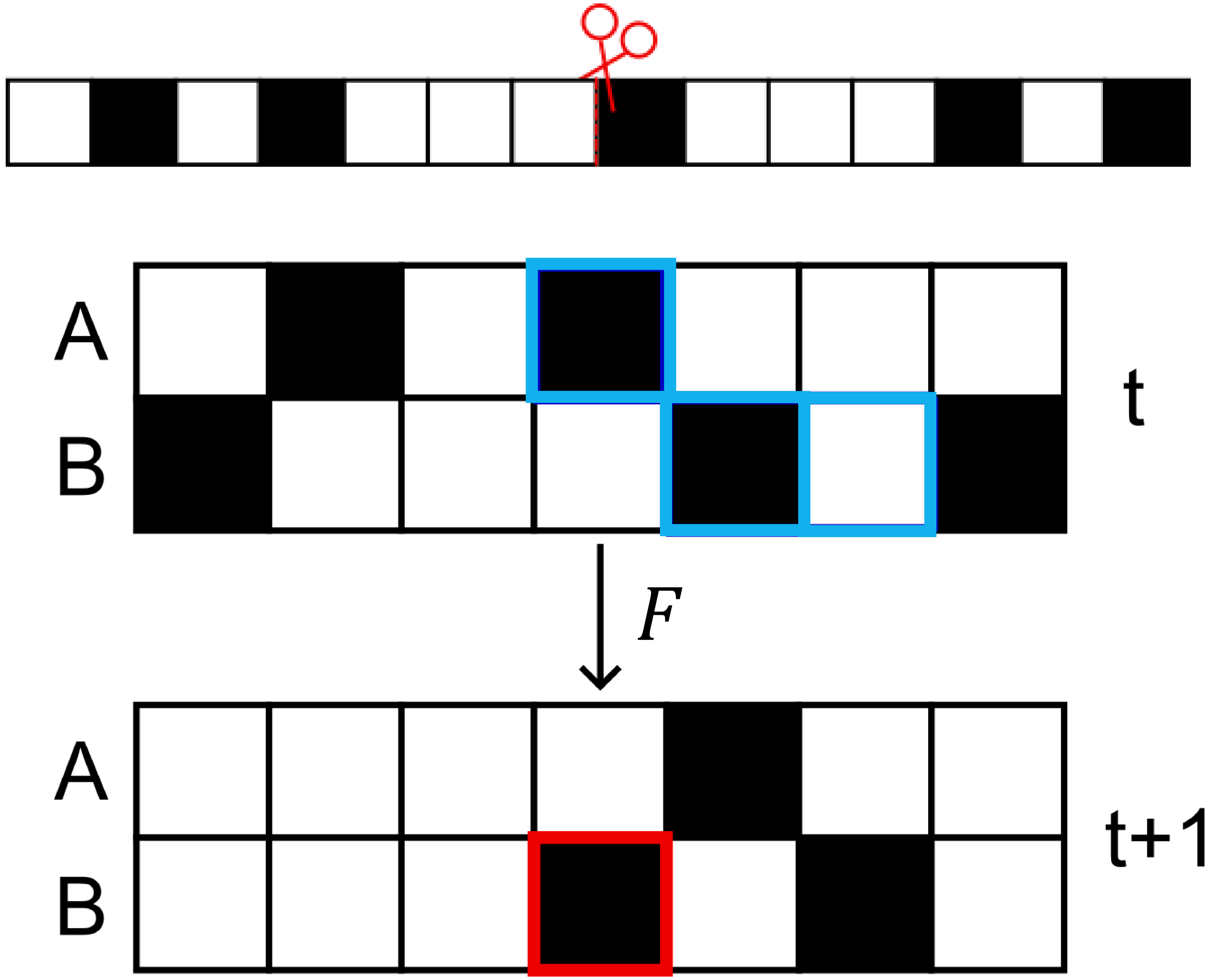
Cannot remove clusters. No good density classifier.

Cellular automata

Two-Line Voting: A better density classifier?

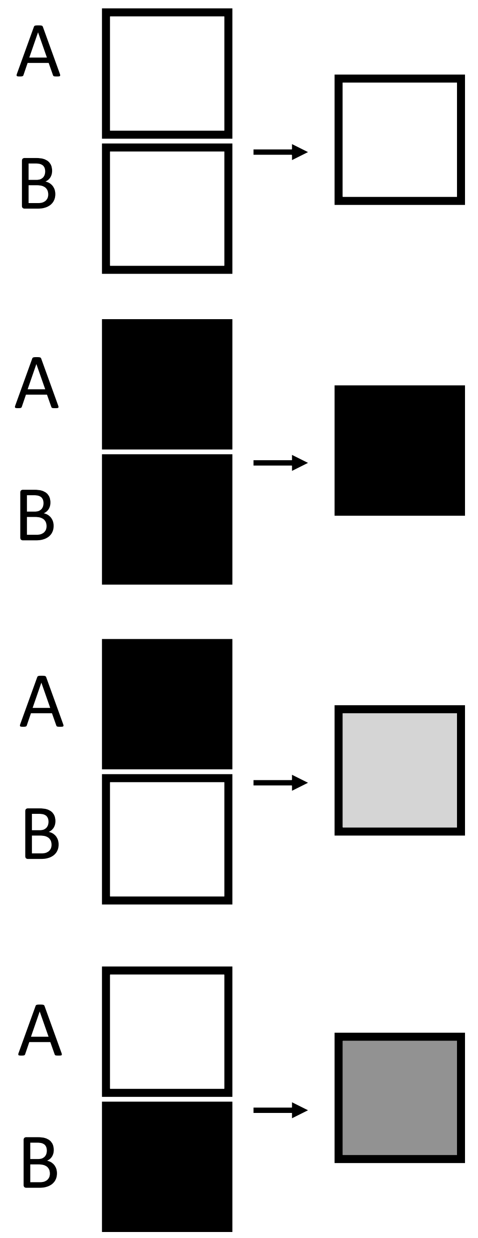


Same local majority rule as 232

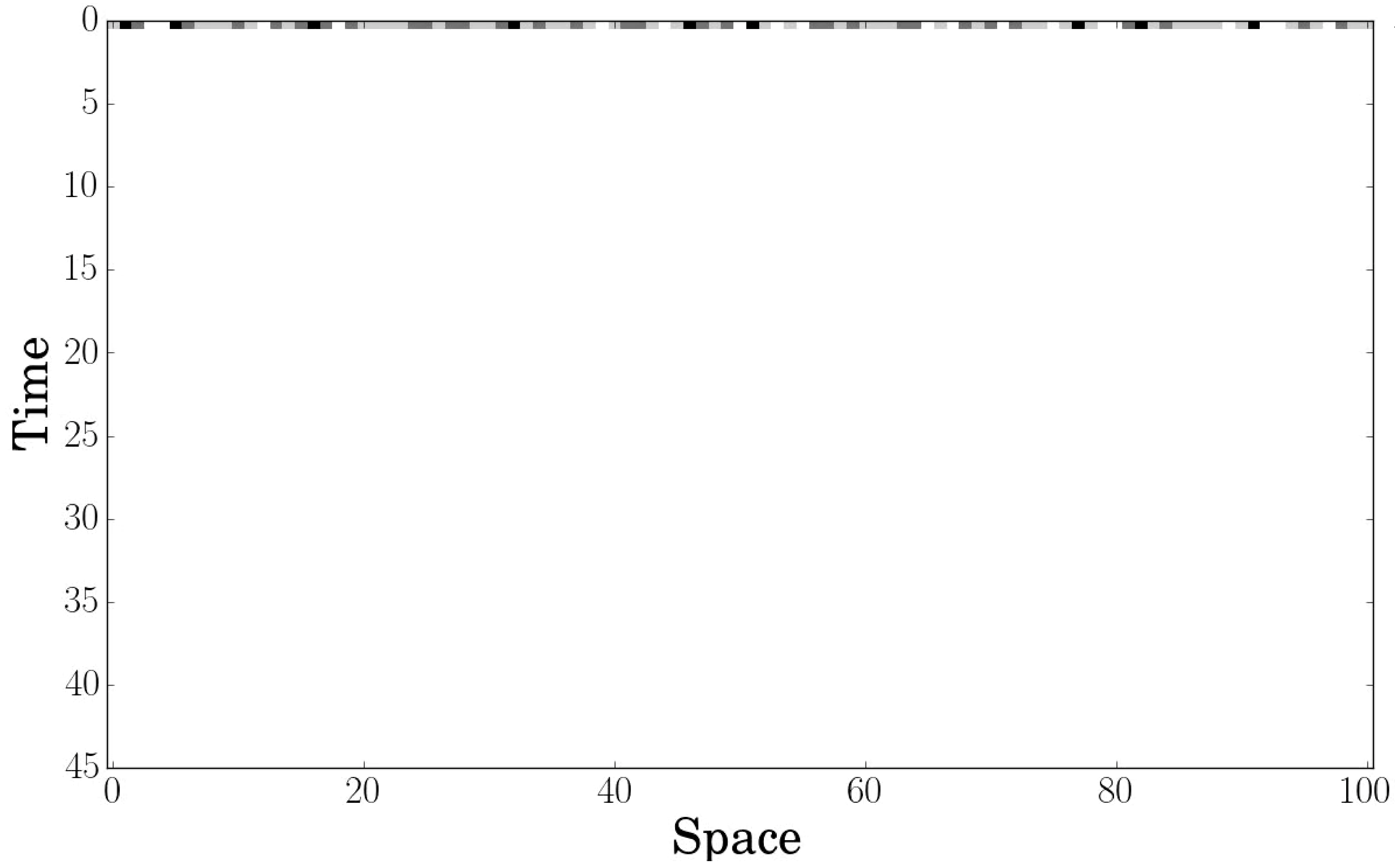


Cellular automata

Color map



Two-Line Voting



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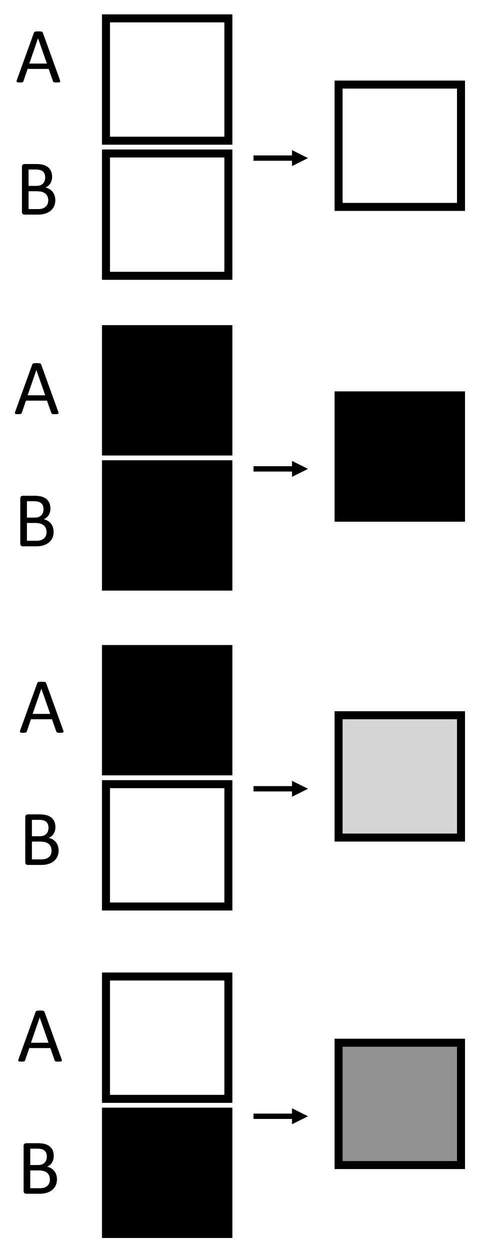

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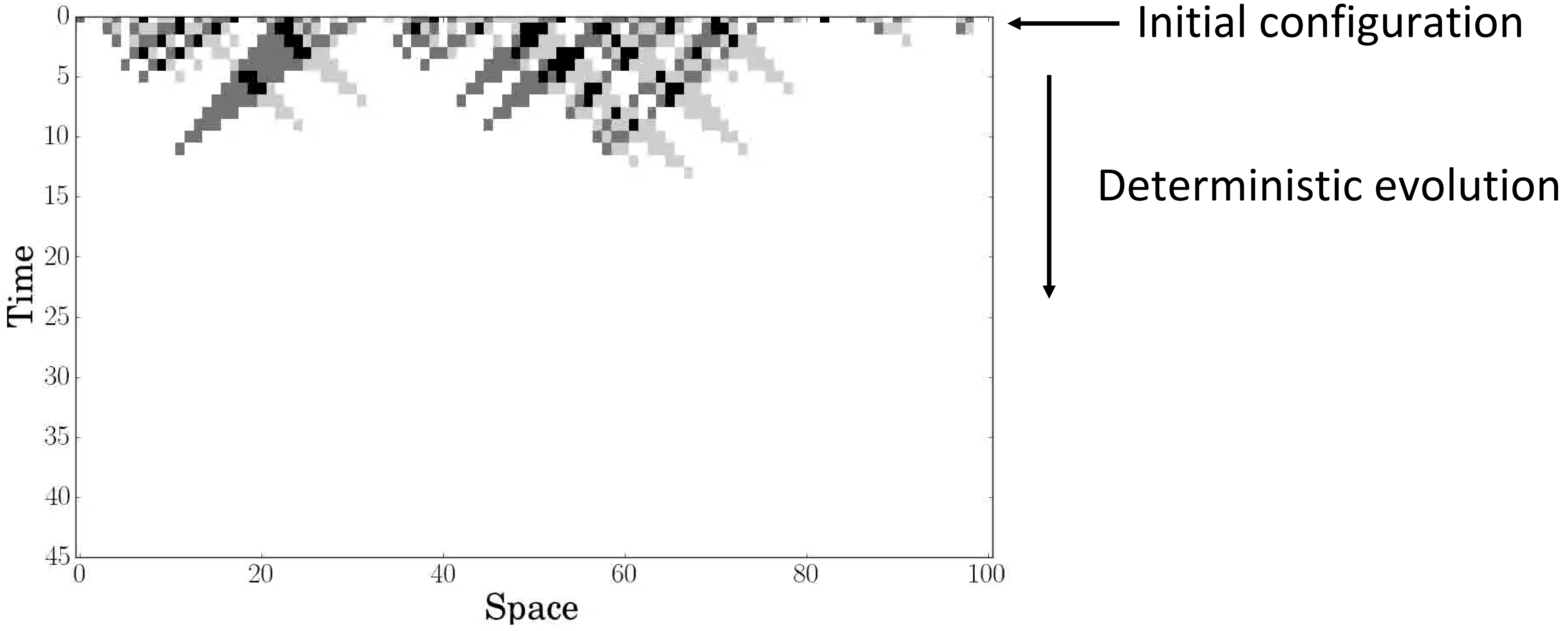

Linear eroder of error clusters. Better density classifier.

Cellular automata

Density classification under continuous noise

- In QEC, we assume every circuit component (gates, measurements,…) is noisy.
  - Application of each global rule subject to noise

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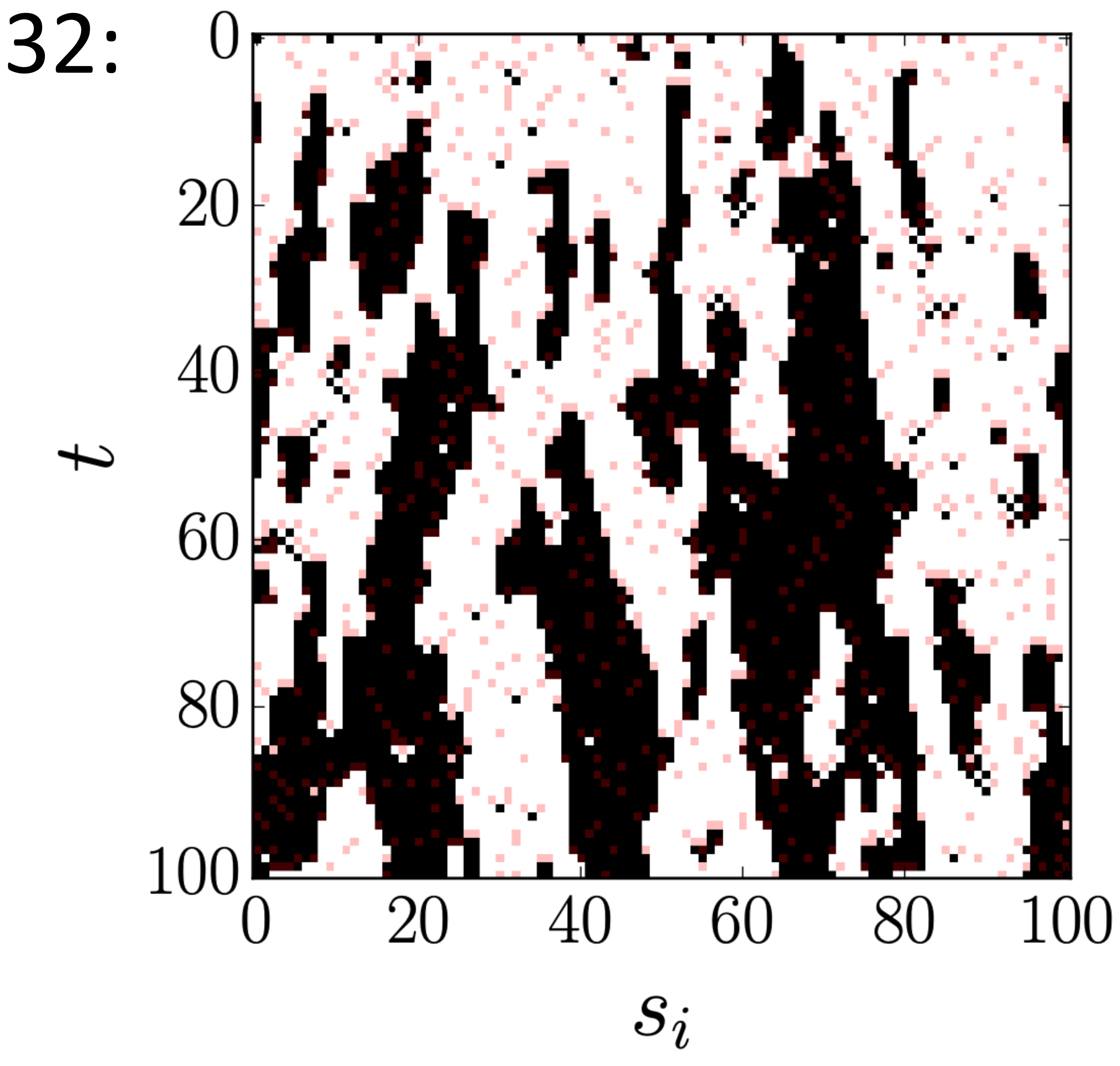
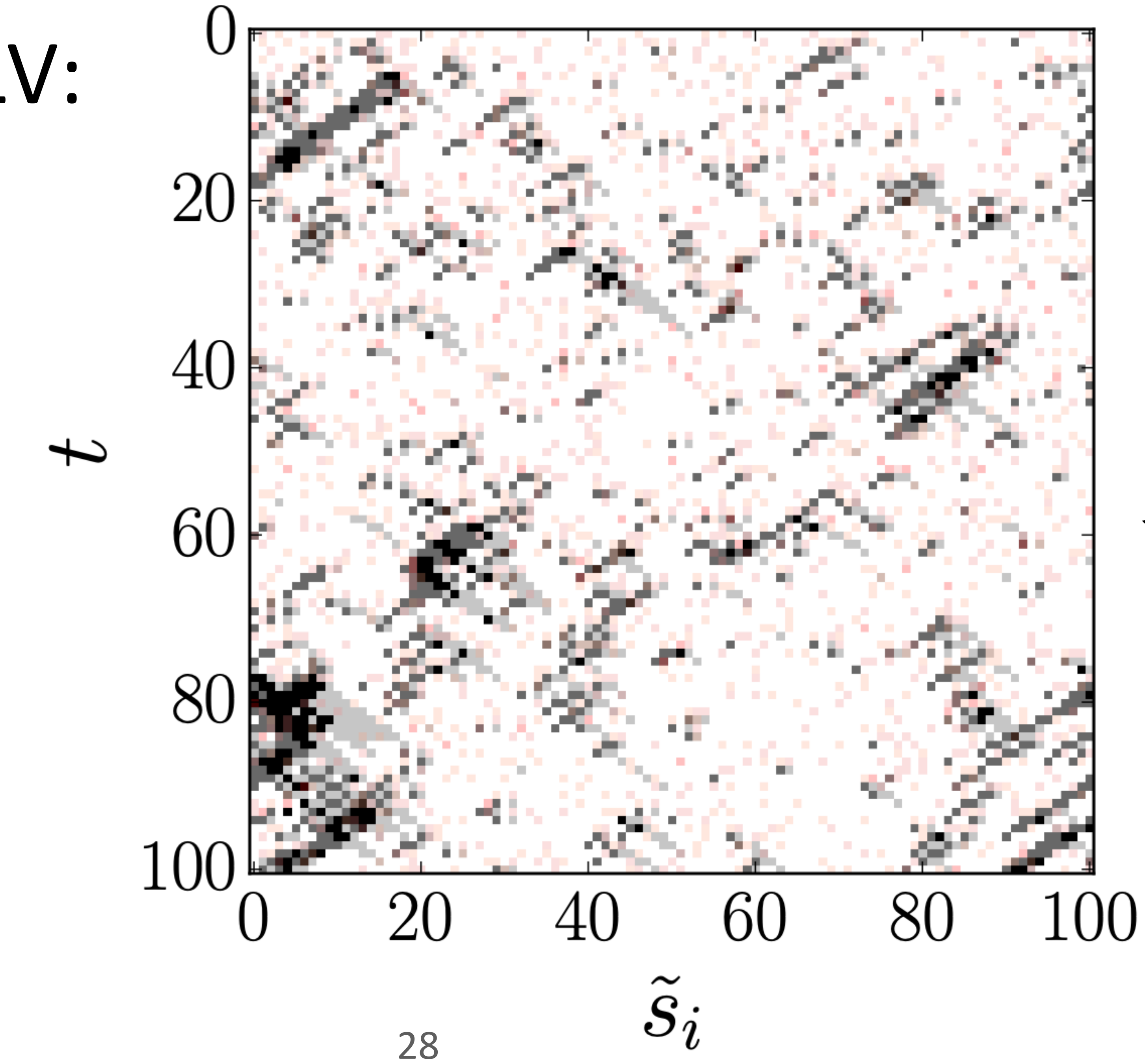
232:



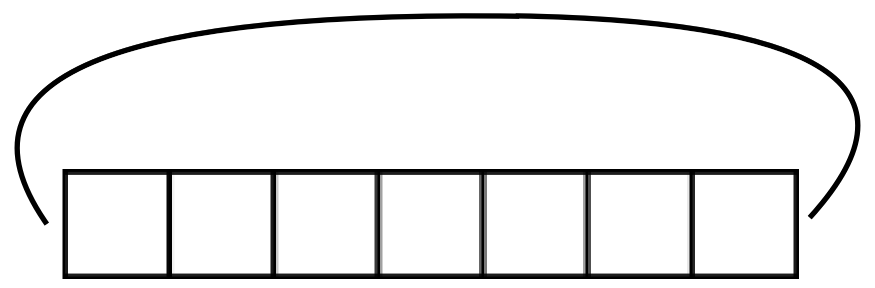
TLV:



Evolved state close to initial state

Quantum cellular automata

Definition



- We define finite, unitary QCA with periodic boundary conditions
  - Related to infinite QCA via wrapping lemma

Quantum cellular automata

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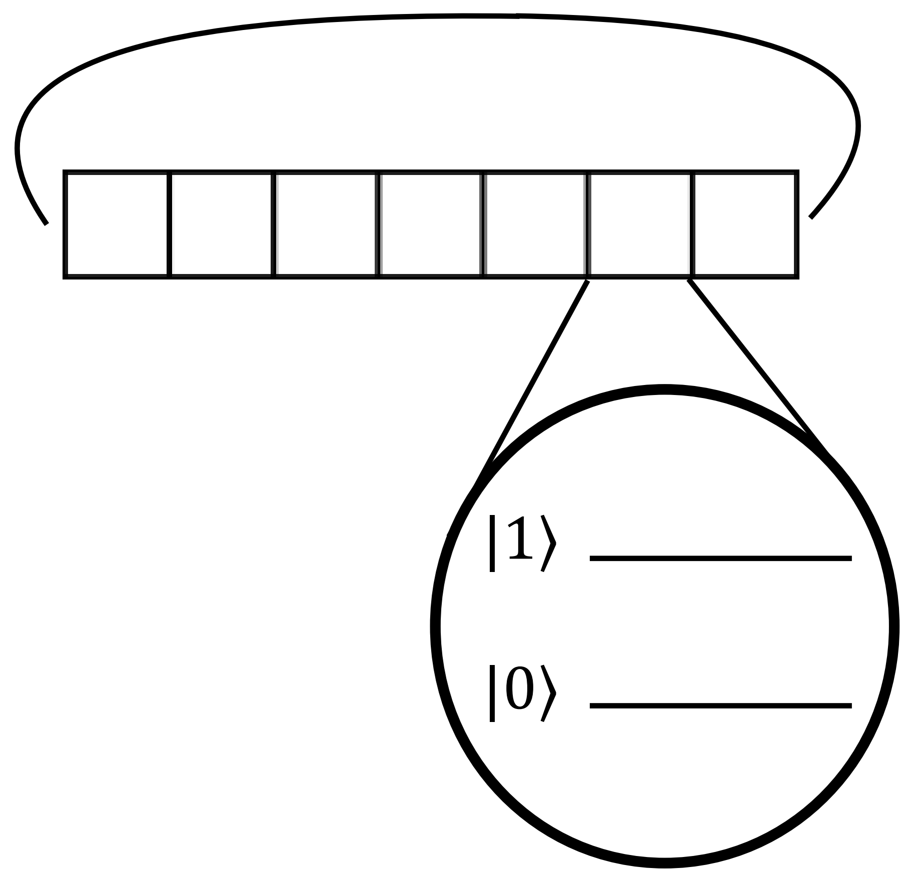

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Quantum cellular automata

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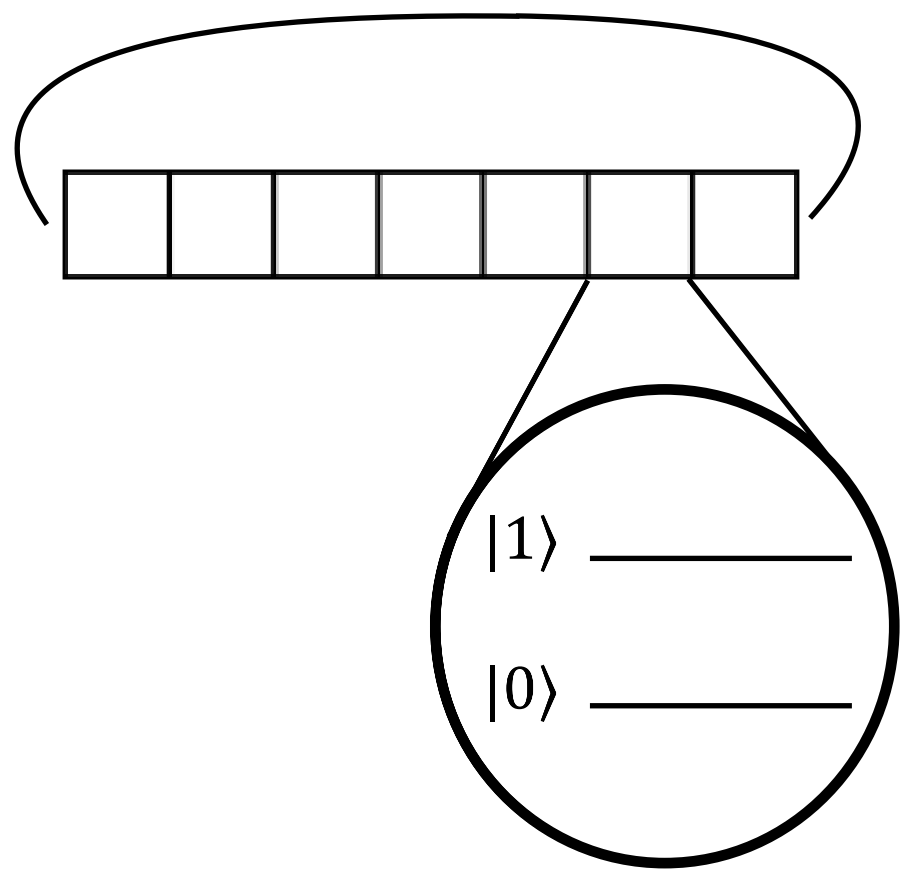

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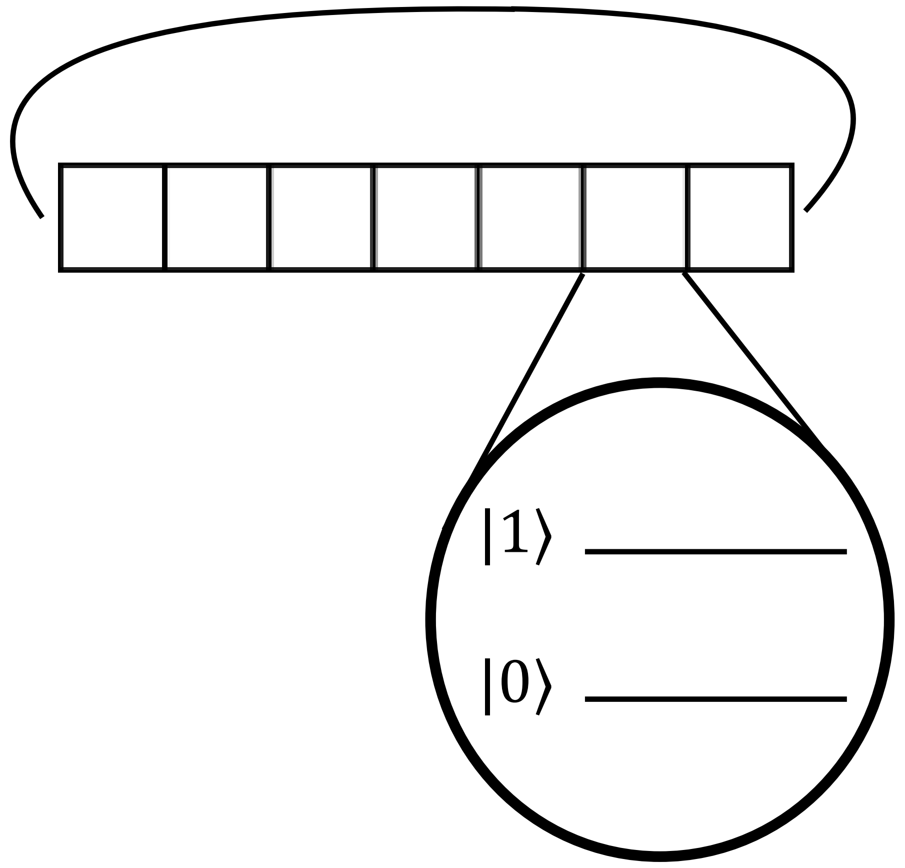

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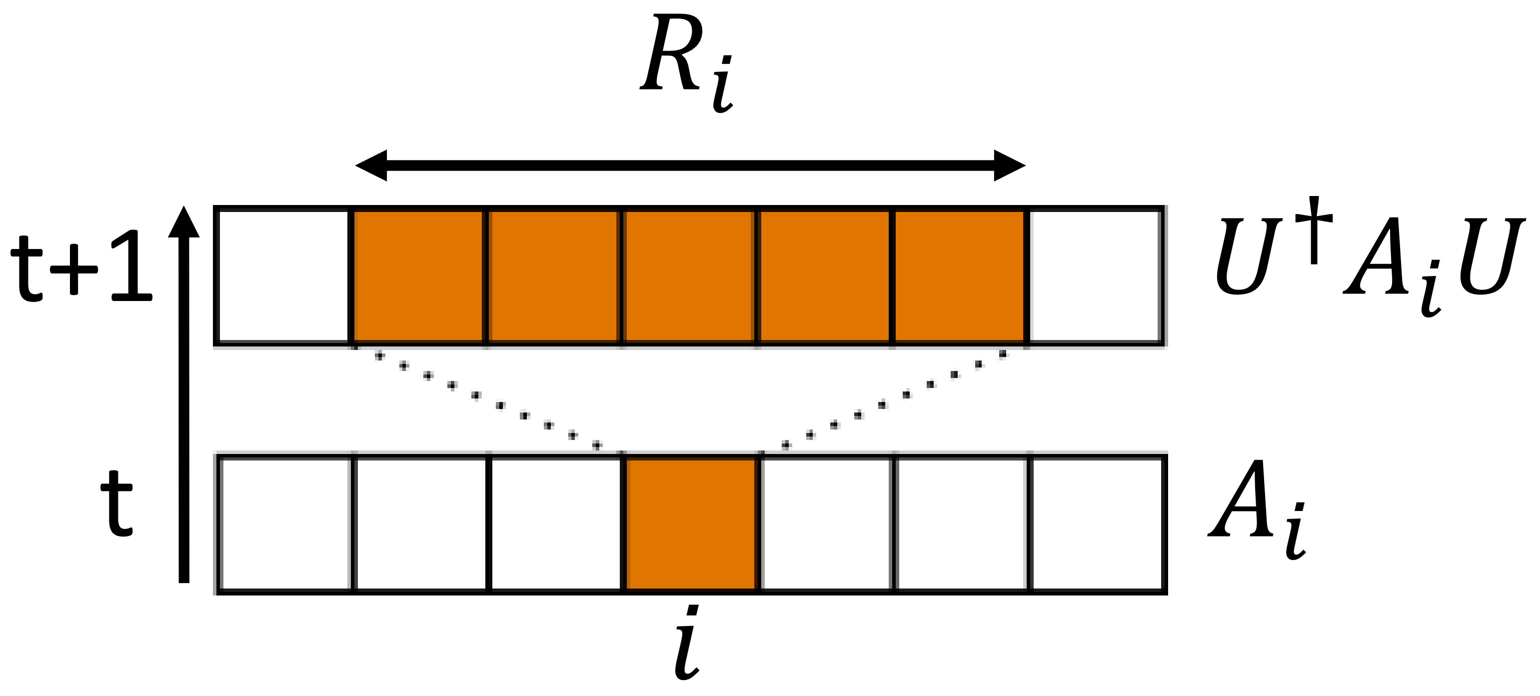
  - Locality-preserving: $u: \mathcal{A}_i \to \mathcal{A}_{R_i}$ for finite region R_i
  - Product of local unitaries $$U = \prod_{i \in L_d} U_i$$



Quantum cellular automata

Quantum version of 232 and TLV

Local majority function:

$$f(a, b, c) = ab \oplus bc \oplus ac$$

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Quantum version of 232 and TLV

flip iff a and b

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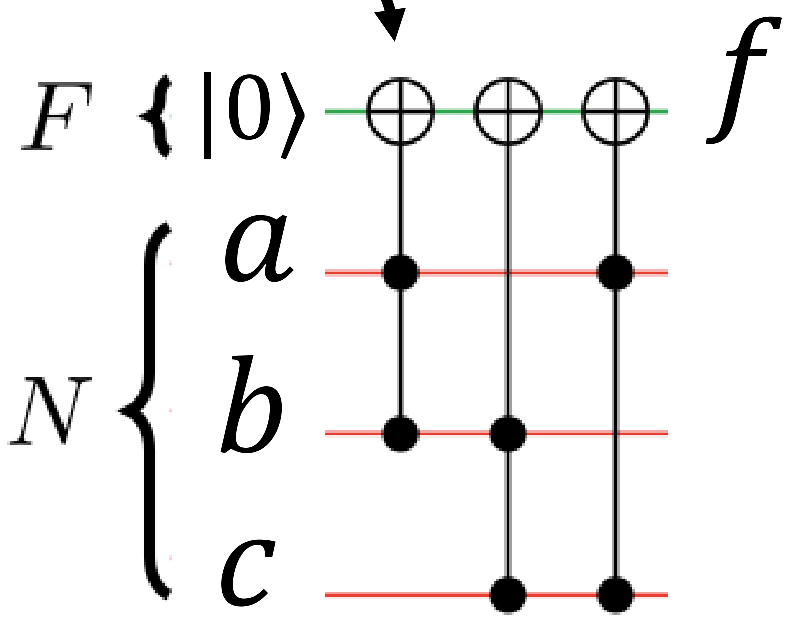
suggests



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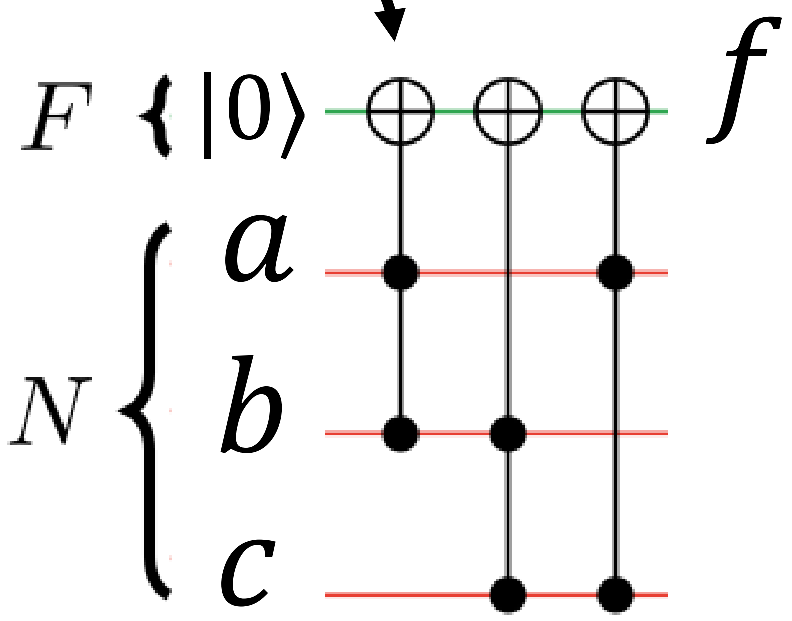

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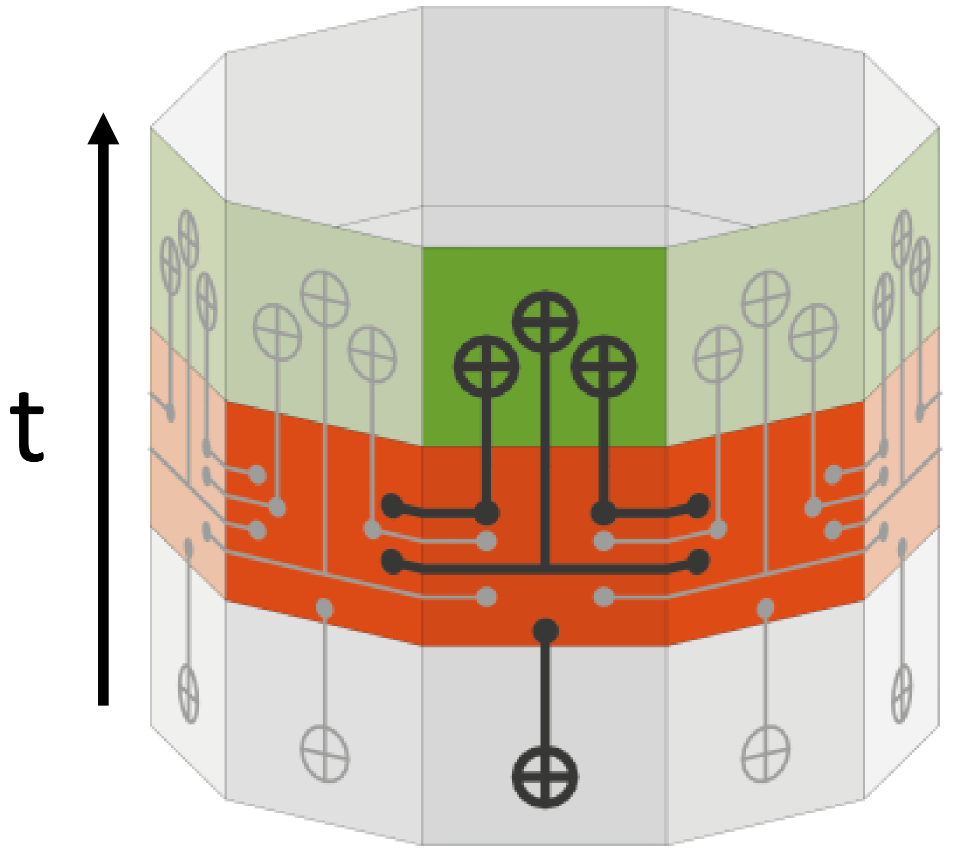
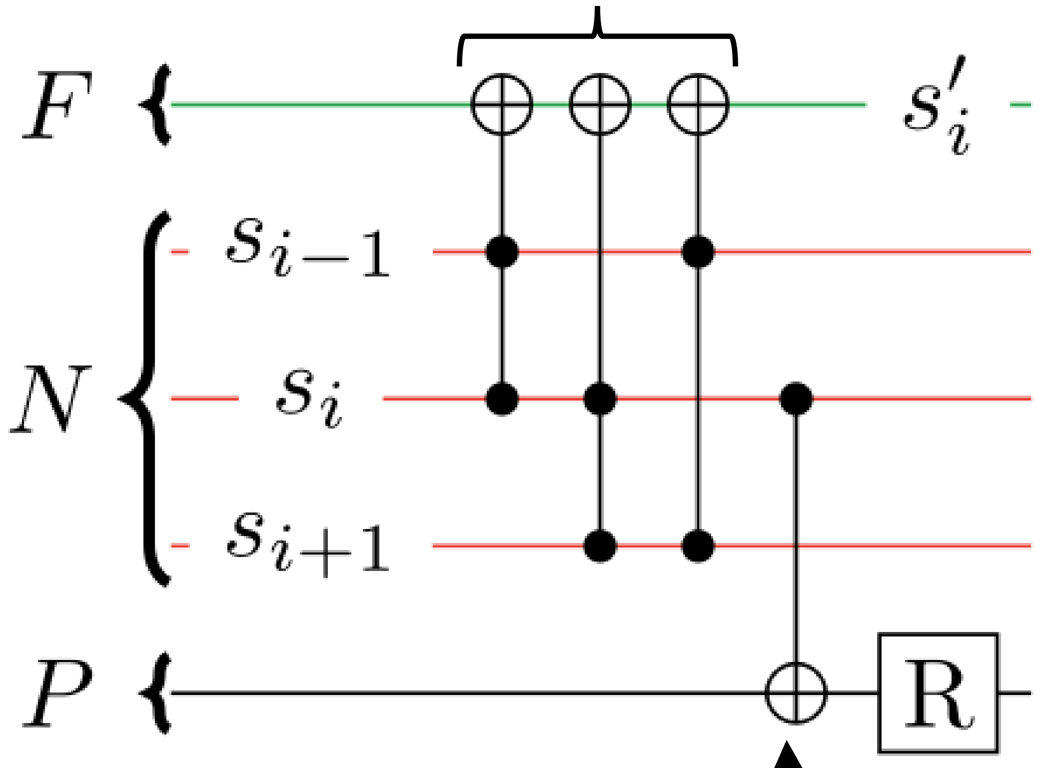


Entangle



Disentangle: Keep code space constant

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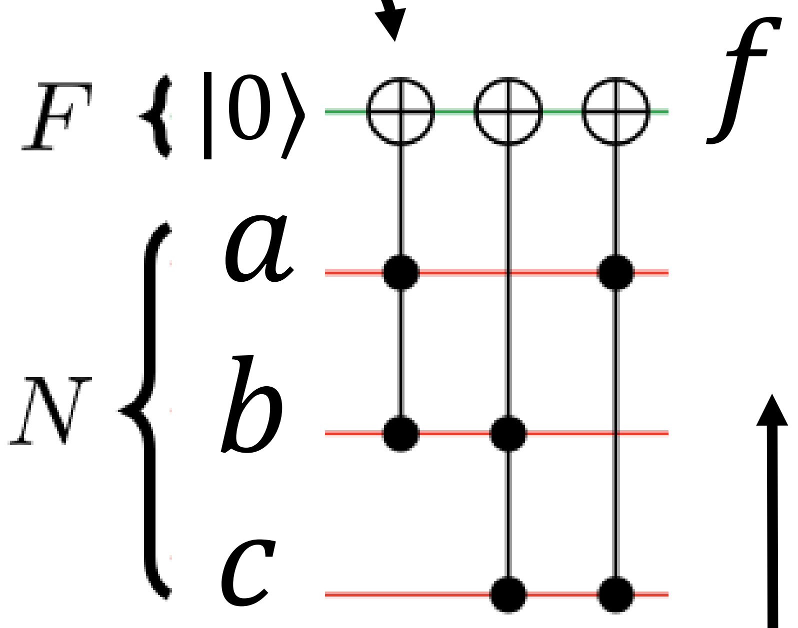
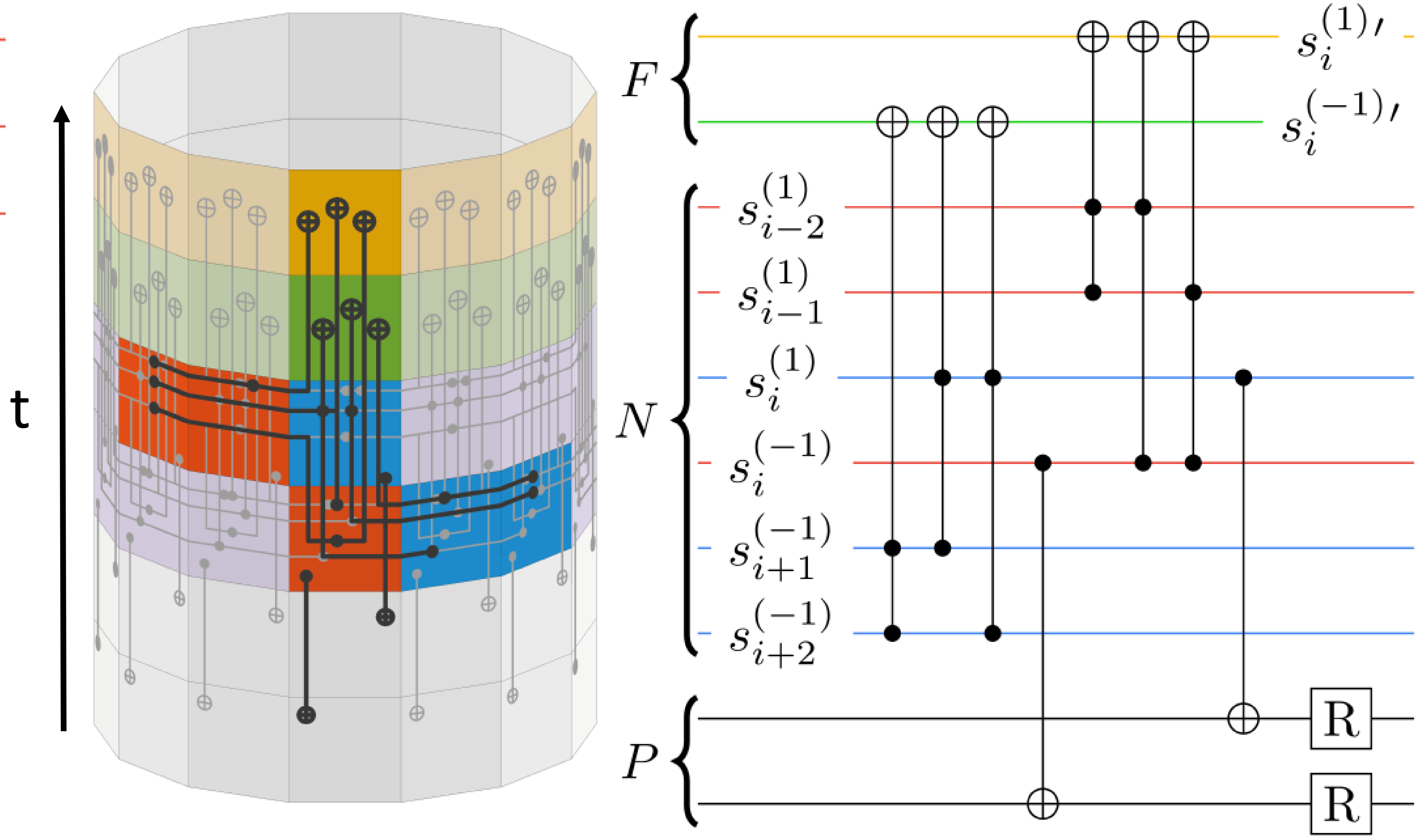


QTLV:



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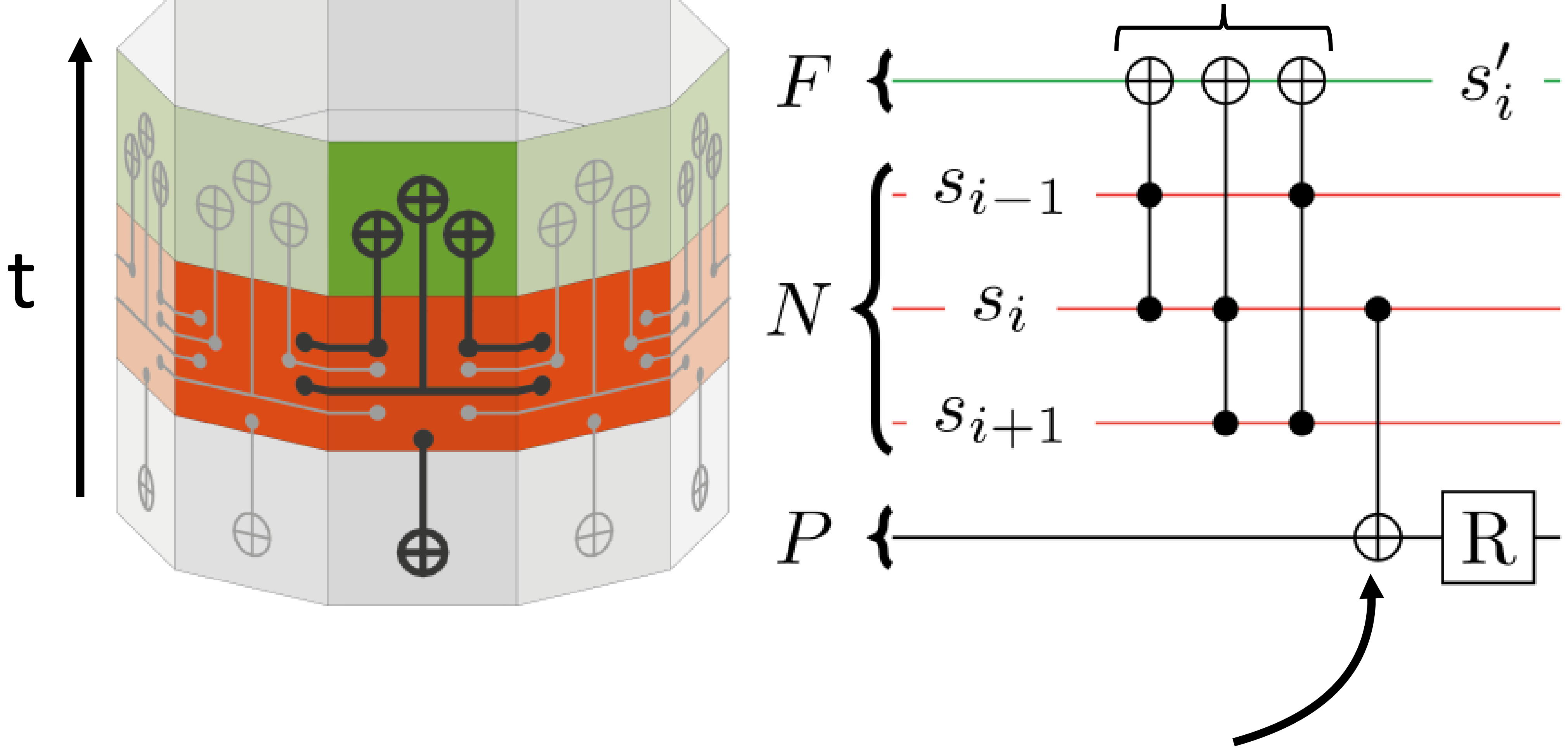

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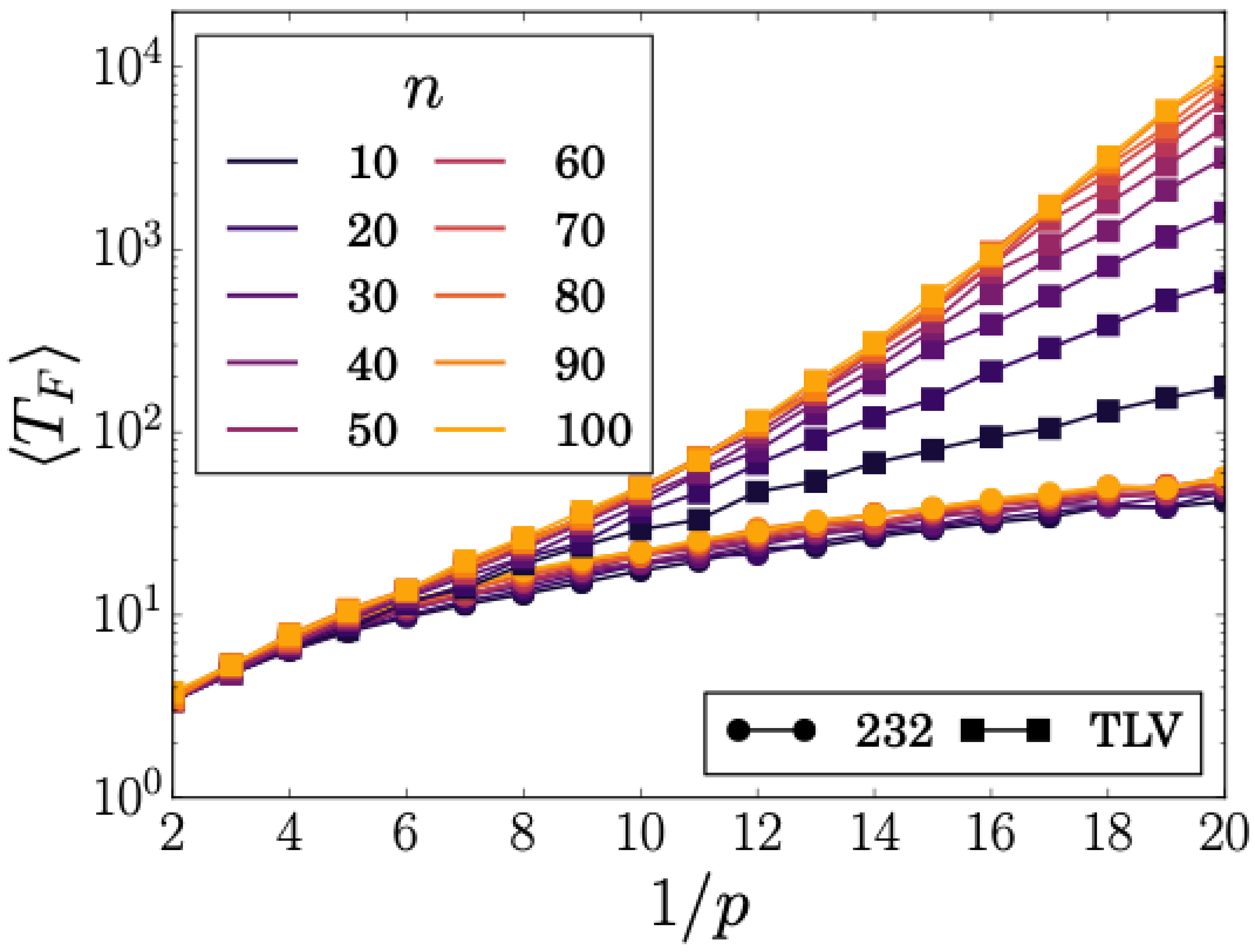
Performance of Q232 and QTLV

Gate noise



Quantum cellular automata

Logical qubit lifetime

$$0:\ \alpha|00000\rangle + \beta|11111\rangle$$
$$\downarrow$$
$$1:\ \alpha|01000\rangle + \beta|10111\rangle$$
$$\downarrow$$
$$\vdots$$
$$\downarrow$$
$$T_F:\ \alpha|01011\rangle + \beta|10100\rangle$$

"**First time** step at which **more than half the qubits** of an initial error-free state **have an error**."

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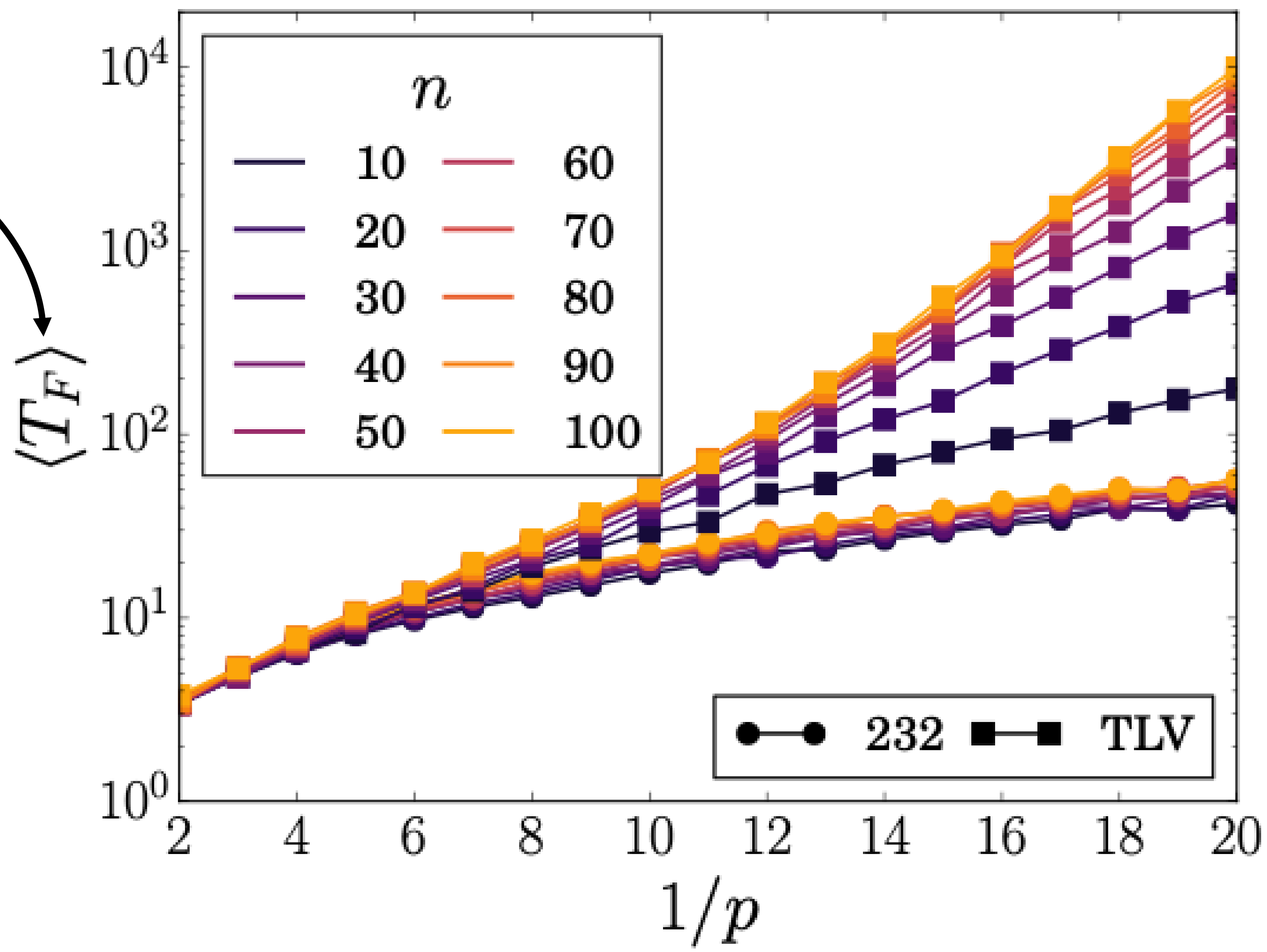

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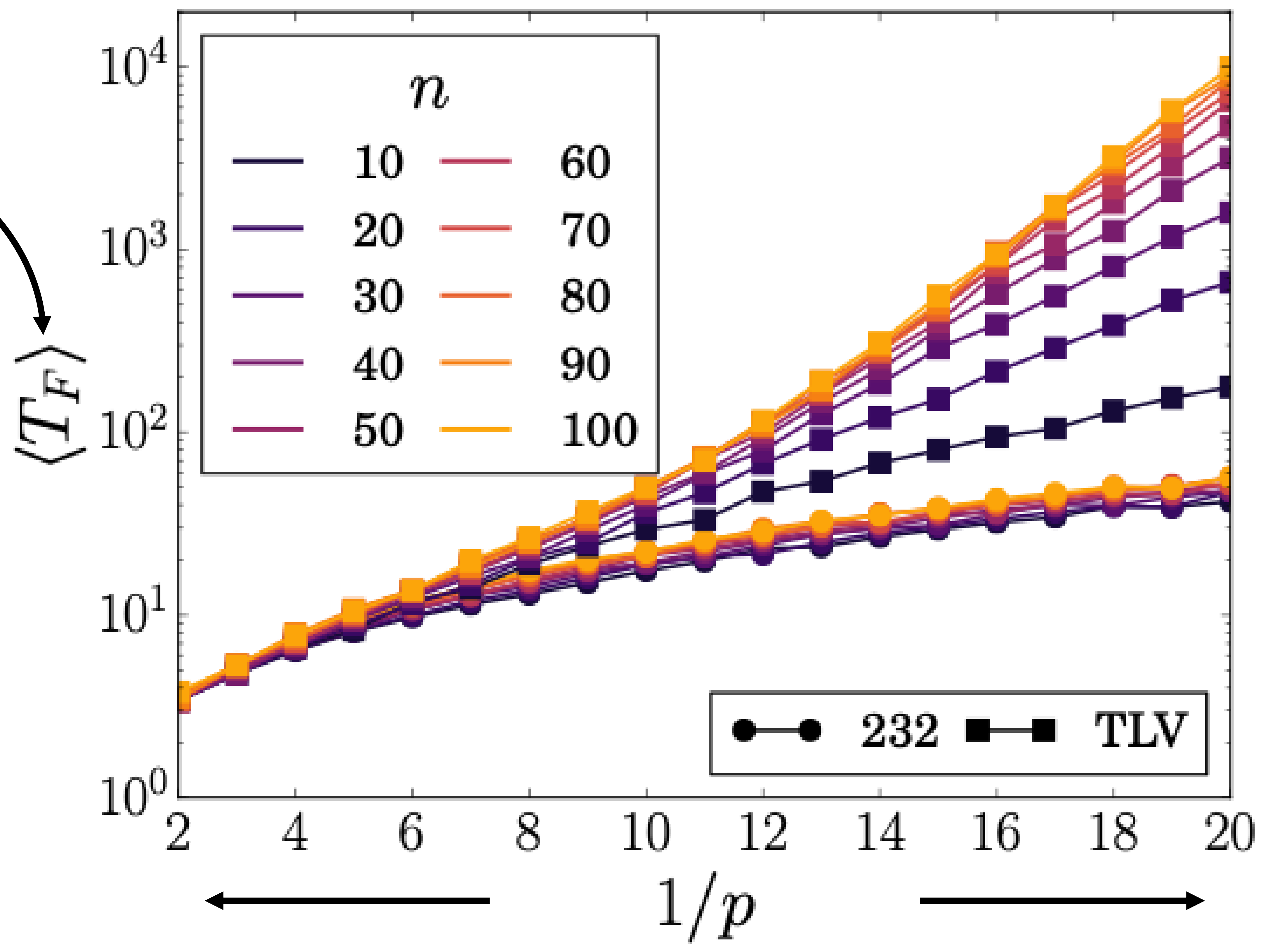

Strong noise

Weak noise

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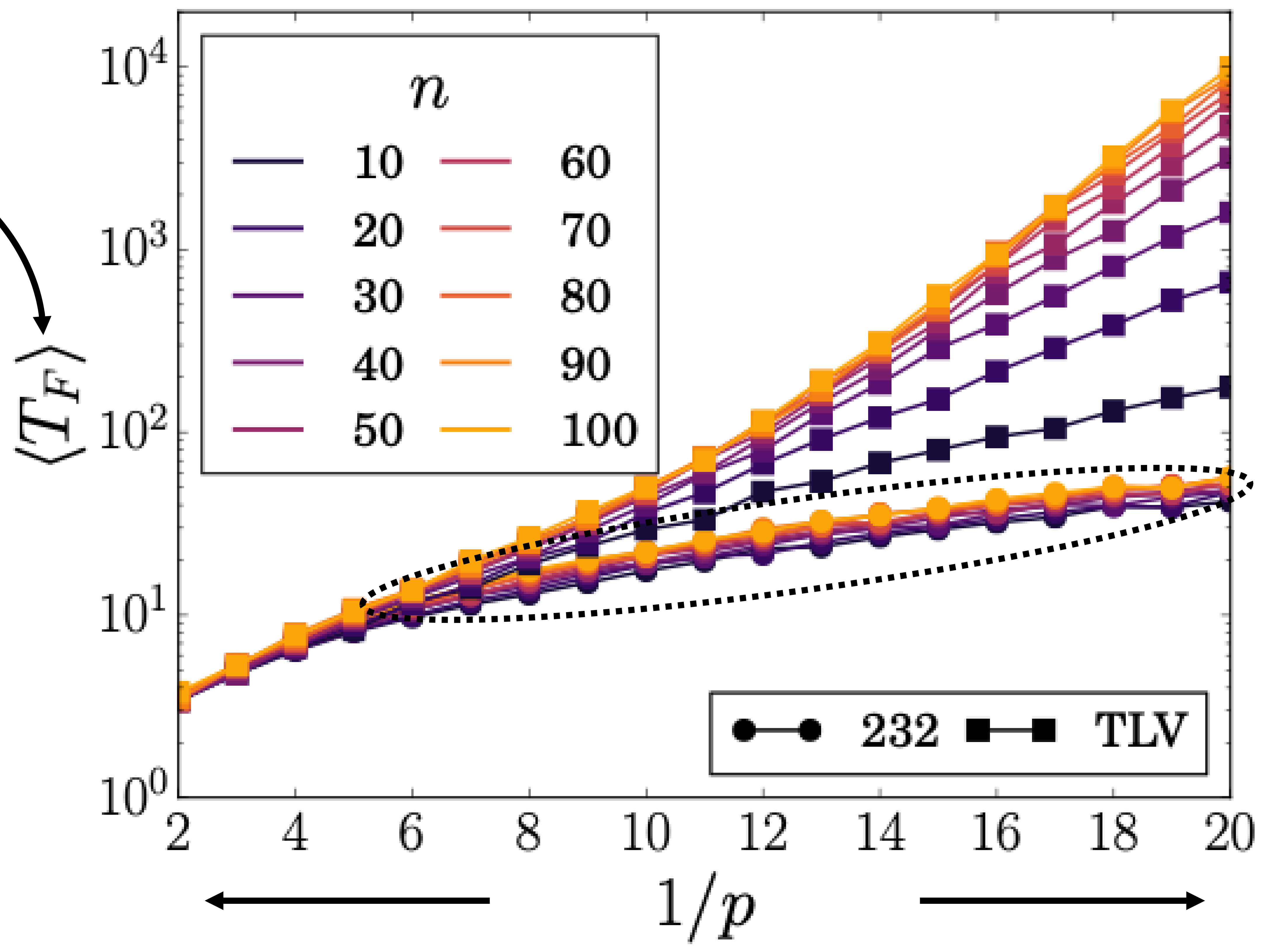

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232: No improvement with system size

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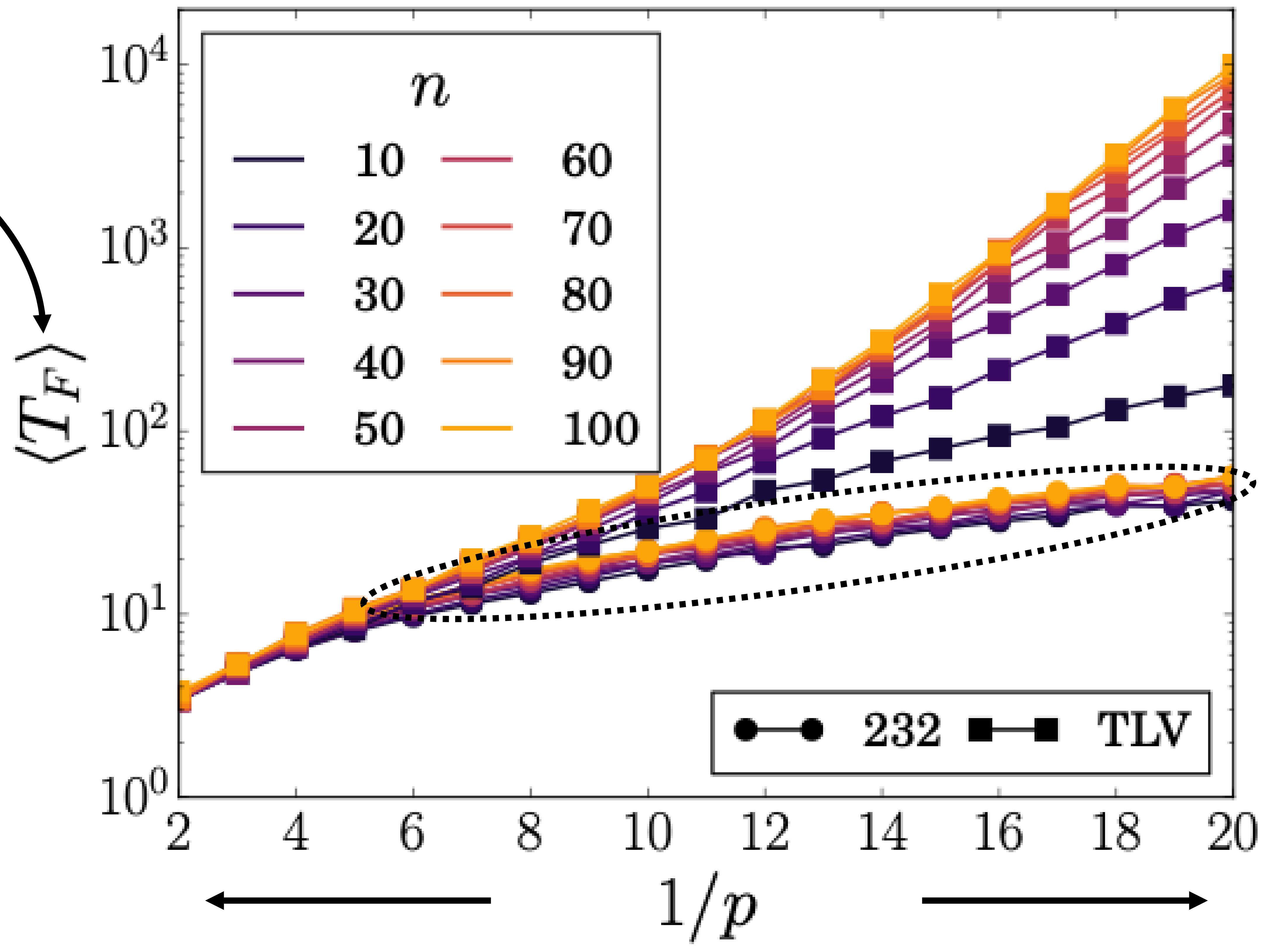


Strong noise

Weak noise

TLV: Better error protection for larger systems

232: No improvement with system size

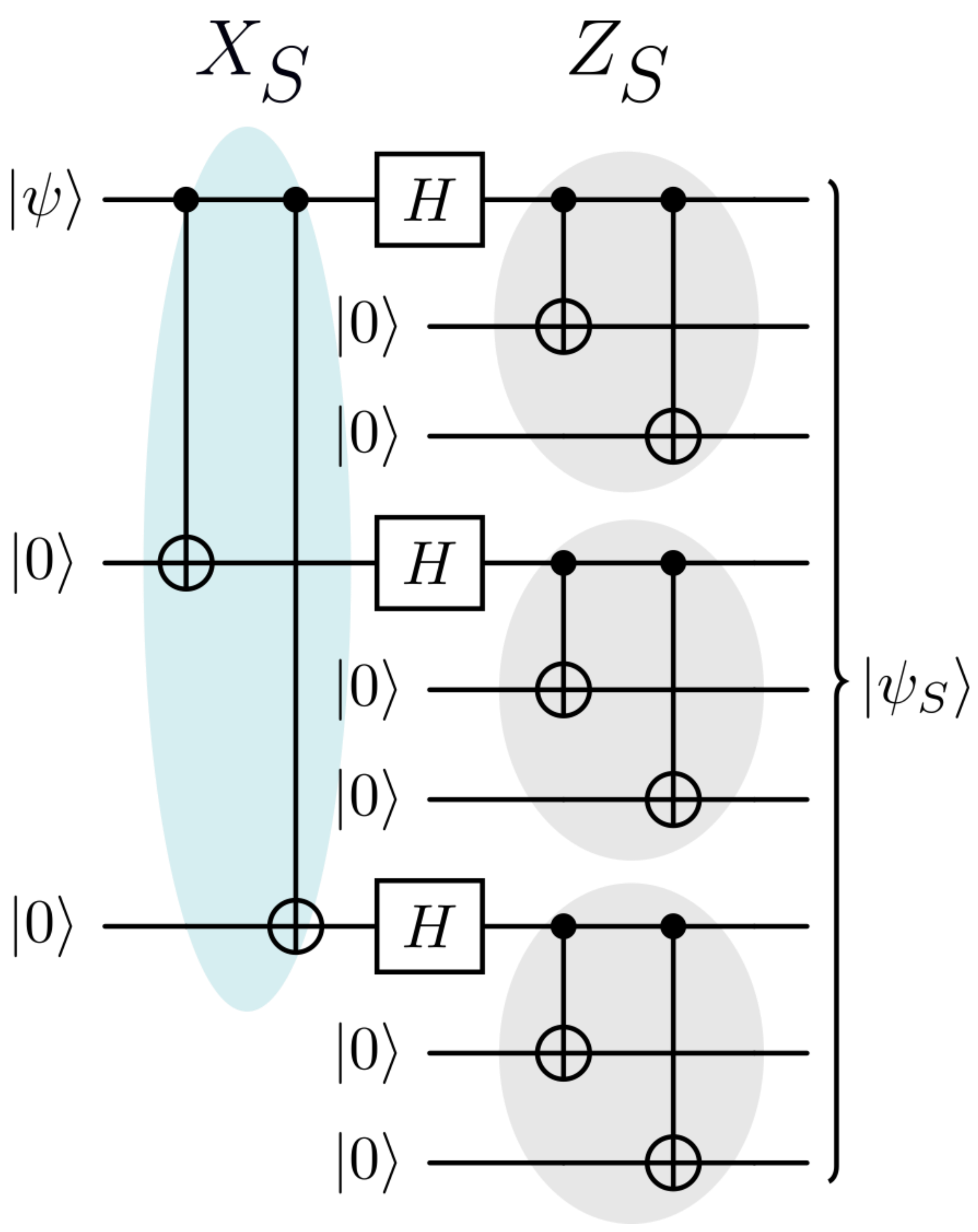
Concatenation

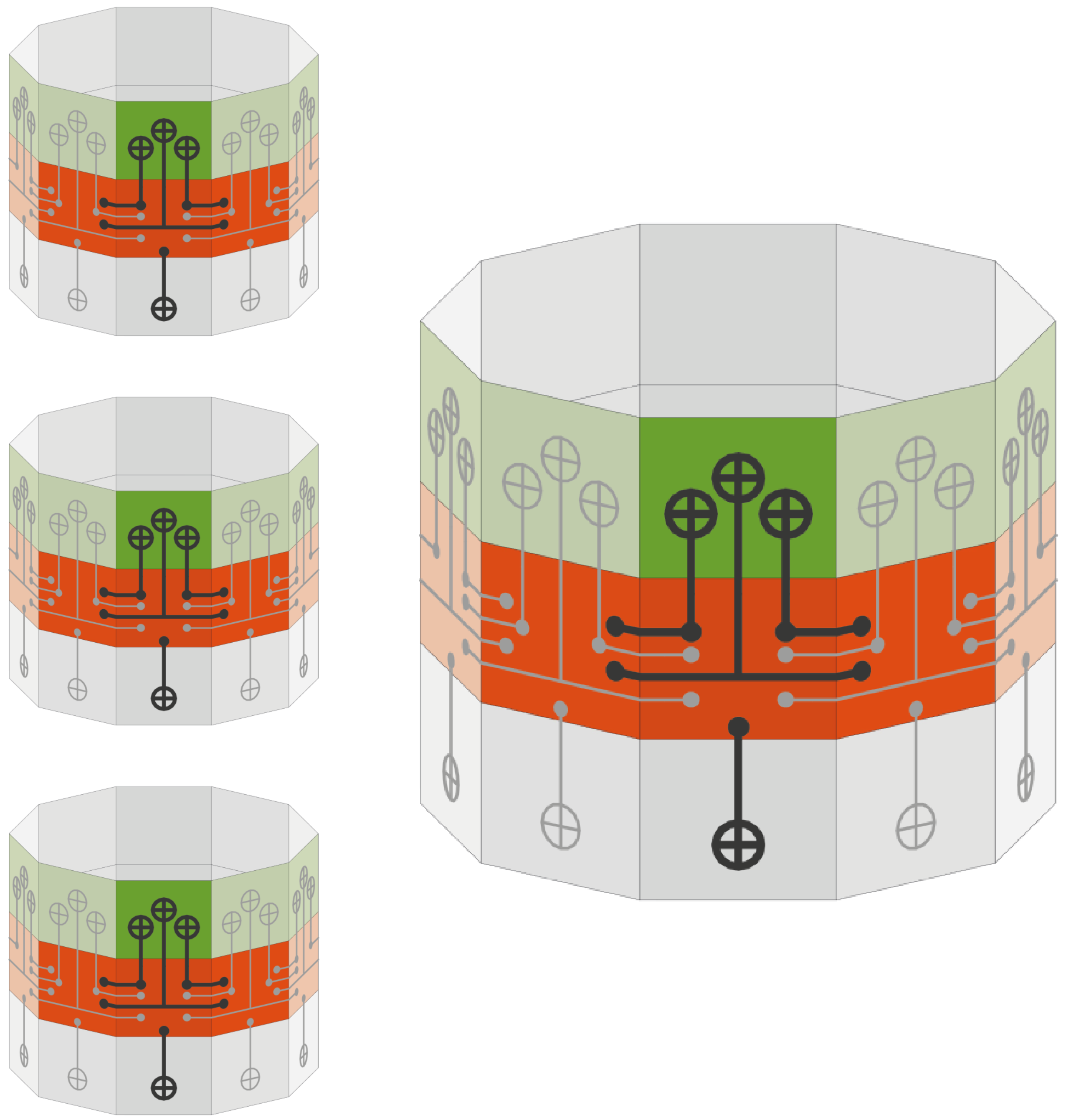
$|0\rangle_L \sim (|000\rangle + |111\rangle)^{\otimes 3}$

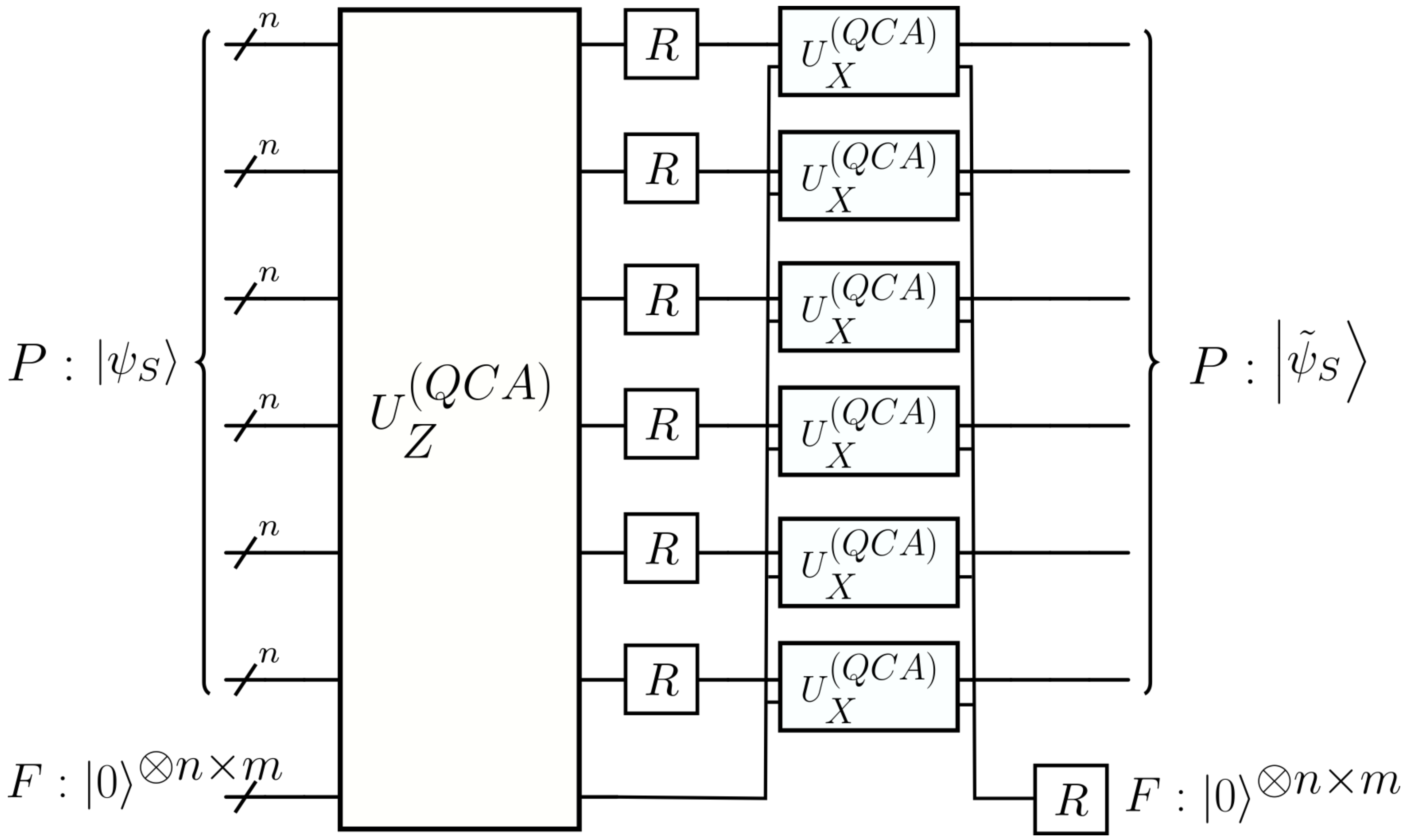
$|1\rangle_L \sim (|000\rangle - |111\rangle)^{\otimes 3}$

Correct bit and phase flip errors

Shor code[1]





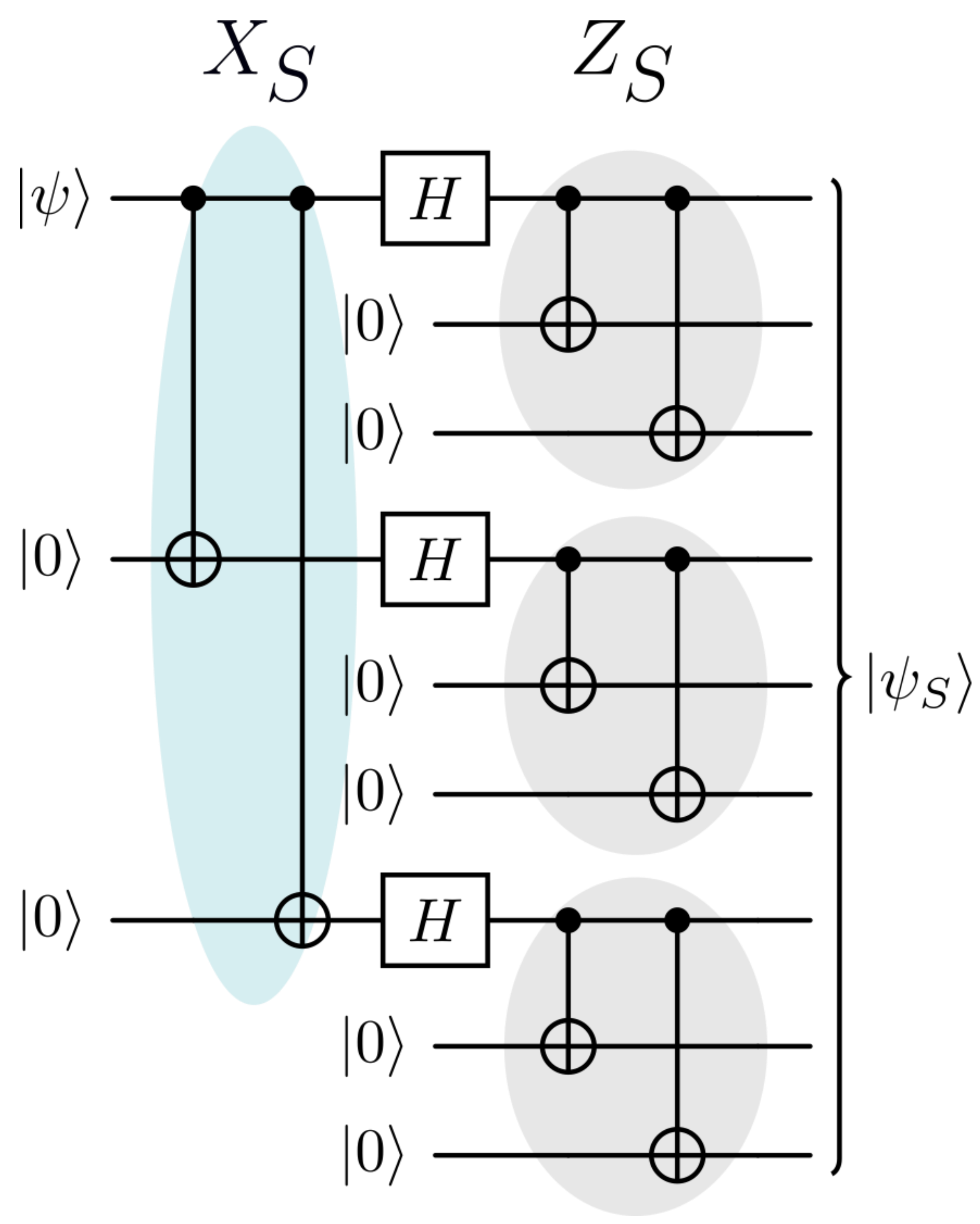


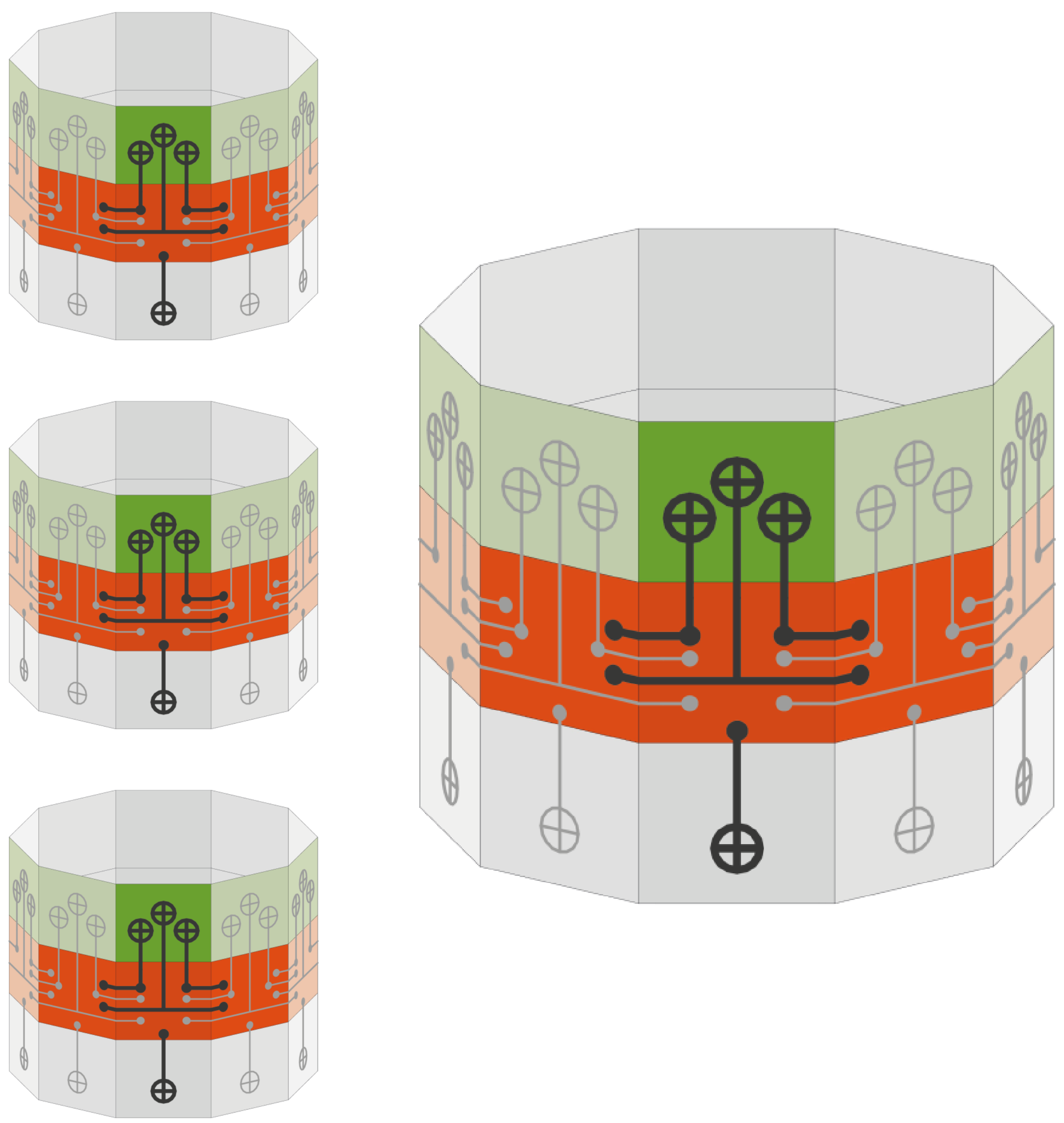
1. A. R. Calderbank, P. Shor, Good quantum error-correcting codes exist, Physical Review A 54, 1098 (1996)

Concatenation

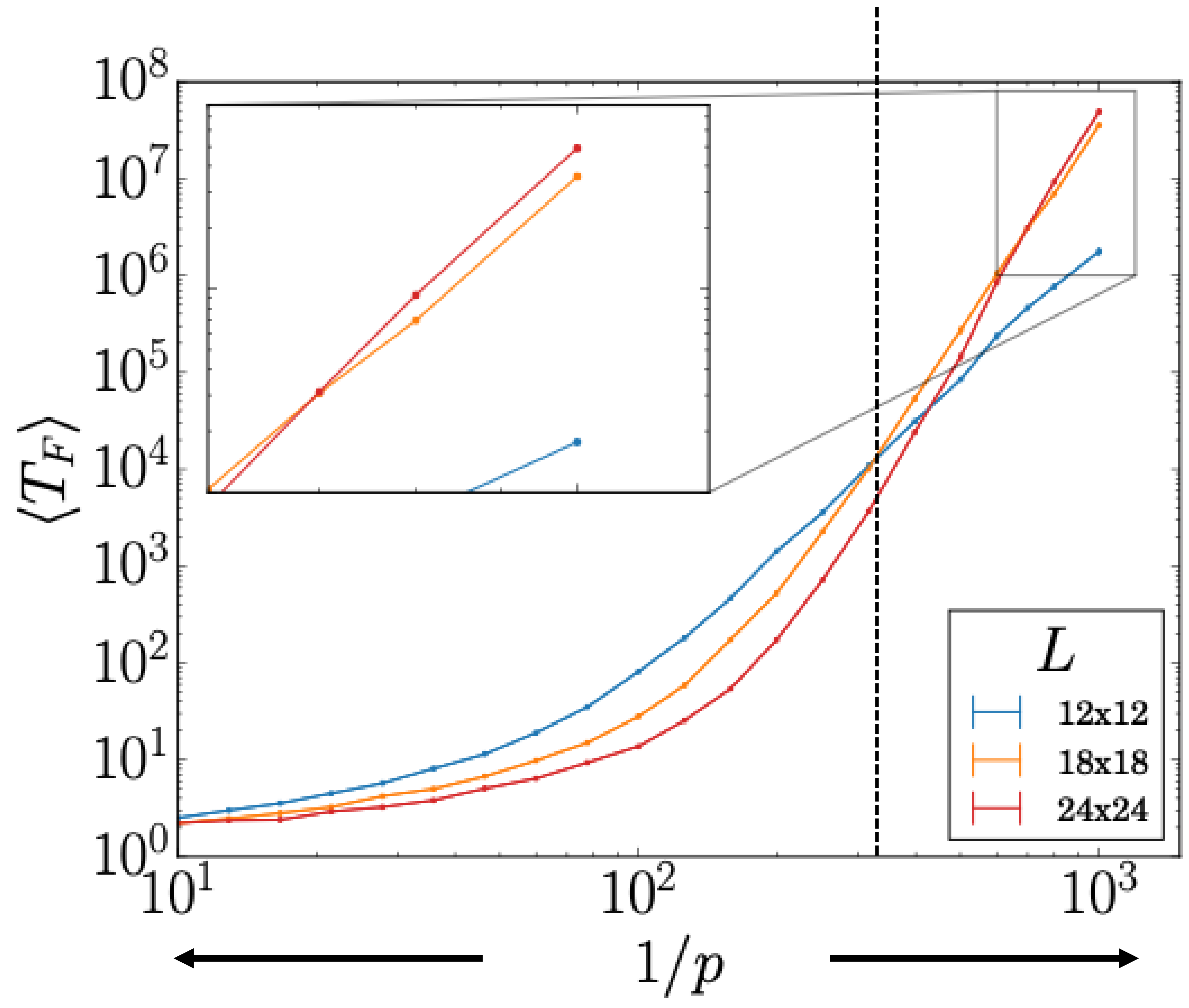
Correct bit and phase flip errors

Shor code[1]





QEC threshold



Strong noise

Weak noise

1. A. R. Calderbank, P. Shor, Good quantum error-correcting codes exist, Physical Review A 54, 1098 (1996)

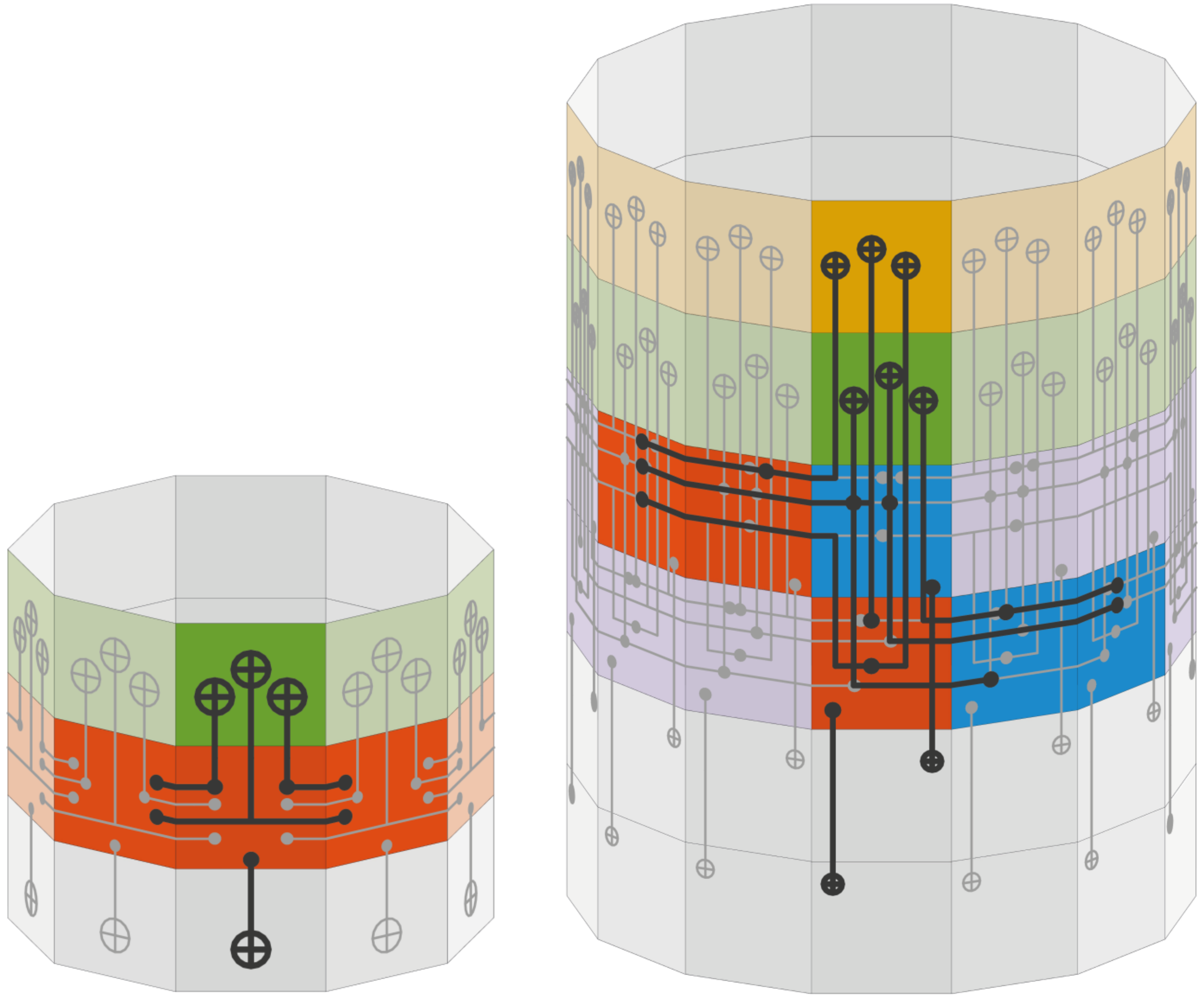
Summary

T. Guedes, M. Müller

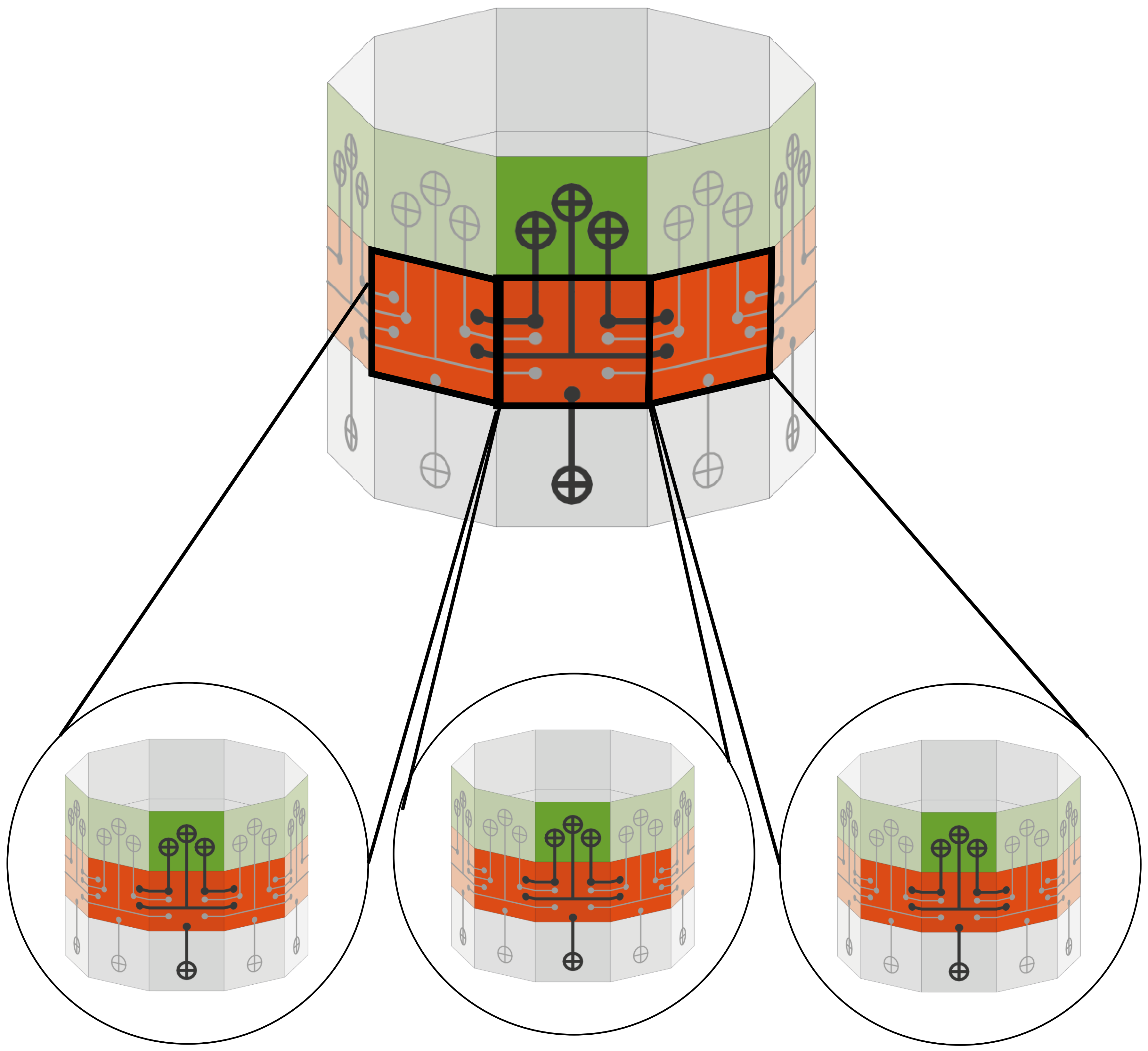
Link to paper

- QCA are local, measurement-free, fully autonomous
  - Attractive for future and near-term experiments
  - Within experimental reach
- Full QEC and QEC thresholds via concatenation

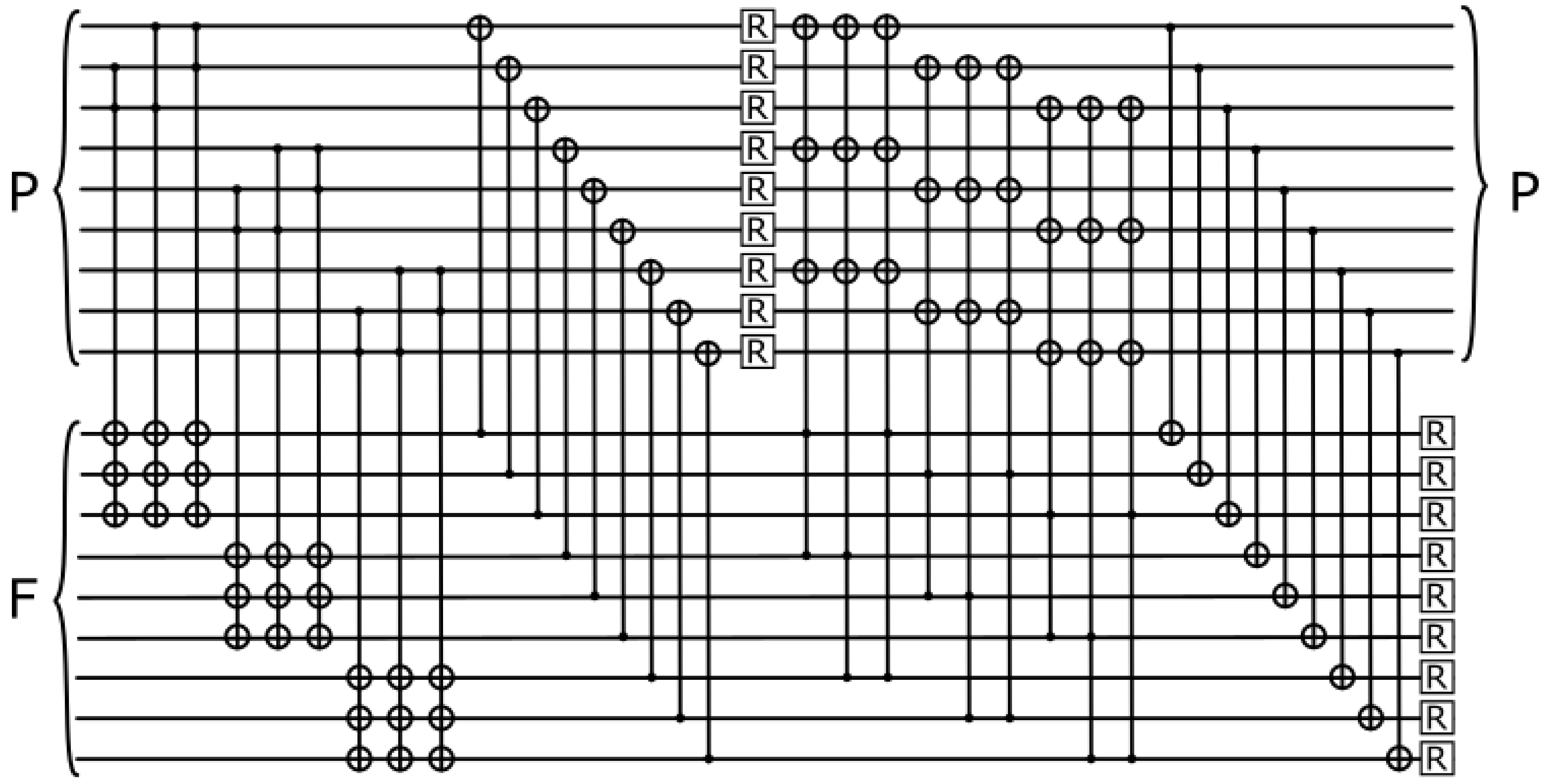
QCA: New paradigm in quantum error correction with many attractive properties for experimental realizations



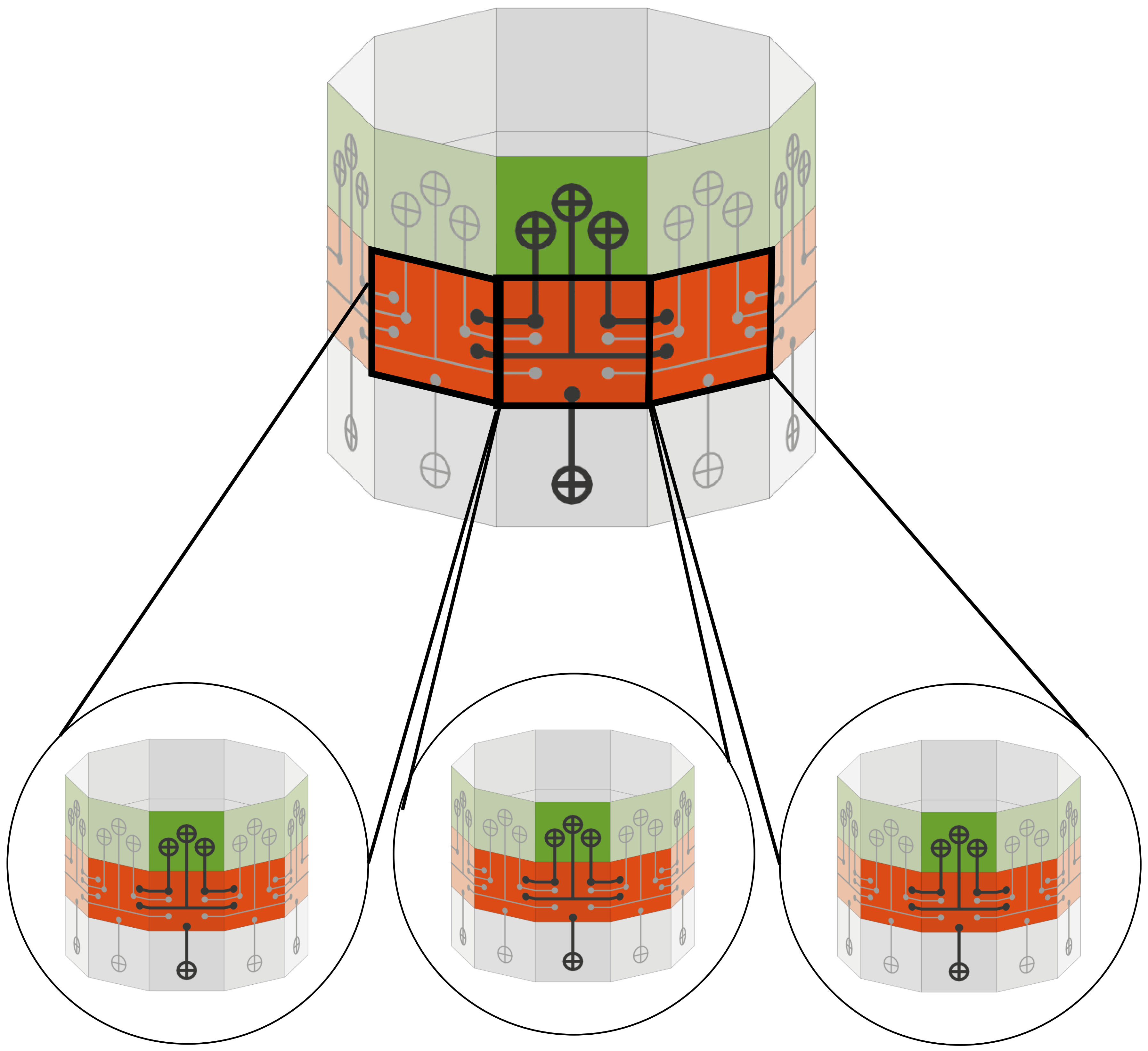
Threshold by concatenation

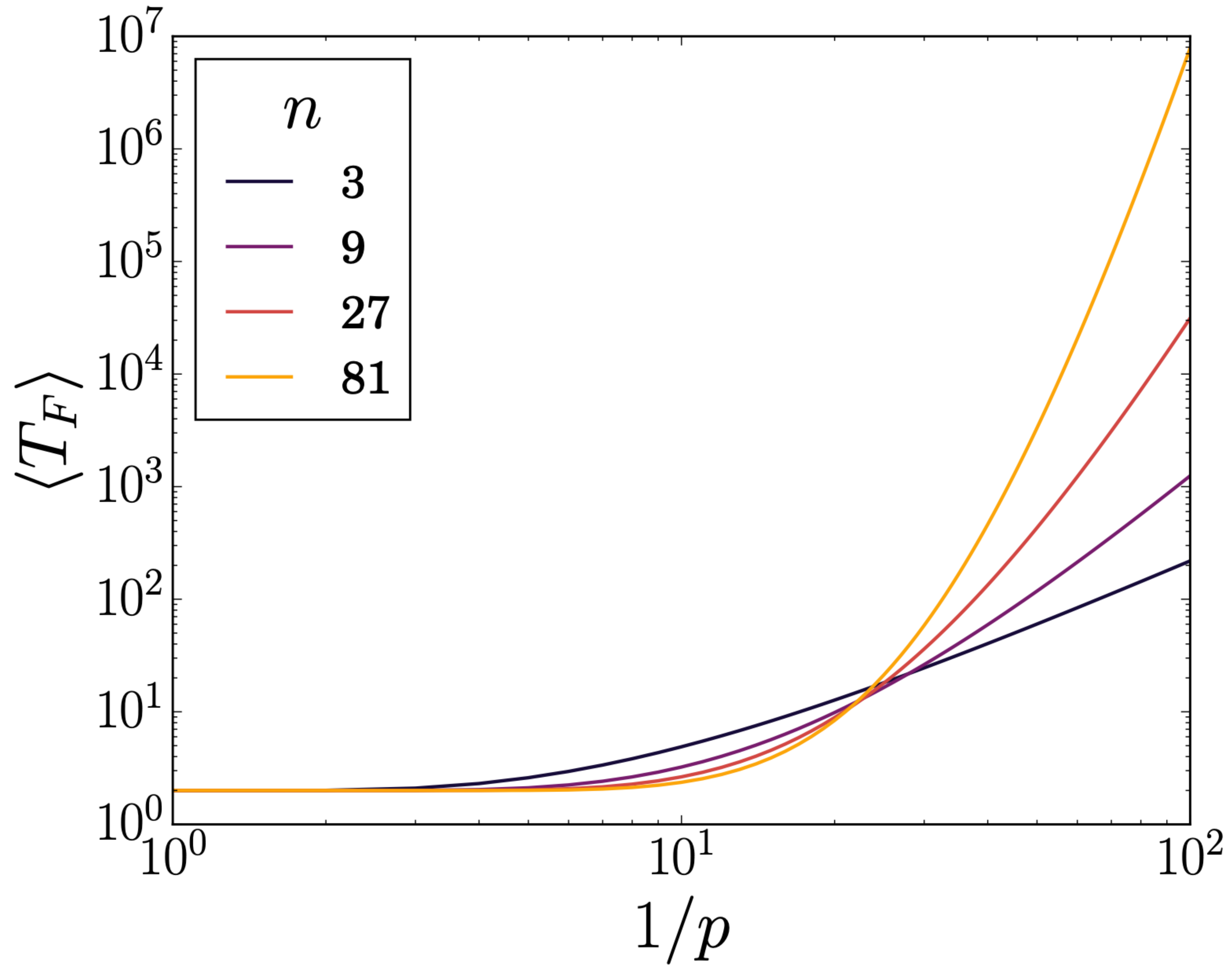


Concatenated QCA



Threshold by concatenation





Q232/QTLV noise model

Gate noise