Experimental Simulation of Symmetry-Protected Exceptional Points with Single Photons

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Kunkun Wang, Lei Xiao, Jan Carl Budich*, Wei Yi*, and Peng Xue*, Physical Review Letters:127.026404 (2021);
Kunkun Wang, Lei Xiao, Haiqing Lin, Wei Yi*, Emil J. Bergholtz*, and Peng Xue*, Sci. Adv. 9, eadi0732 (2023);
Kunkun Wang, J. Lukas K. Konig, Kang Yang, Lei Xiao, Wei Yi, Emil J. Bergholtz*, and Peng Xue*, arXiv: 2410. 08191 (2024).

Outline

- Background: Exceptional Points
- Exceptional Non-Hermitian Metals
- Higher-Order Exceptional Points and Monopole
- Summary

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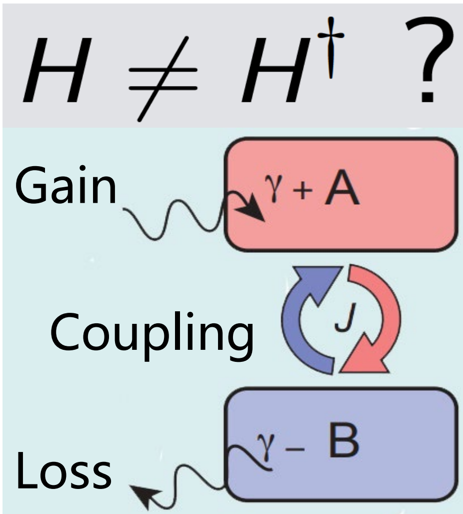
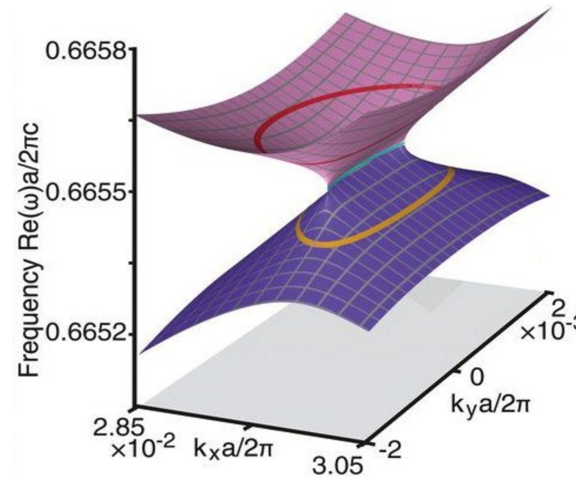
Exceptional Points





H. Zhou, et, al., Science 359, 1009 (2018)

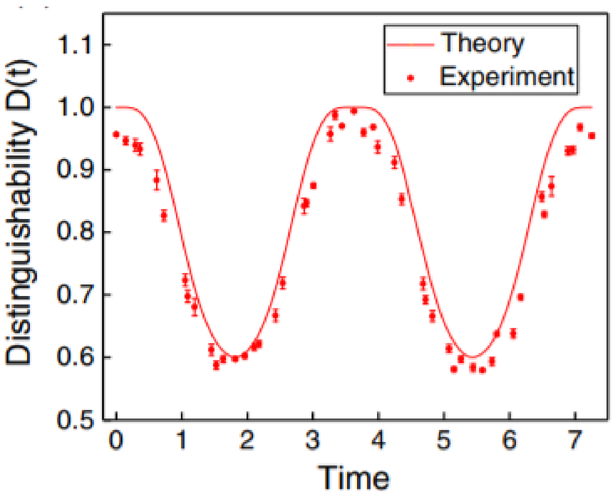
Spectral properties of non-Hermitian Hamiltonian

- Complex eigenvalues,

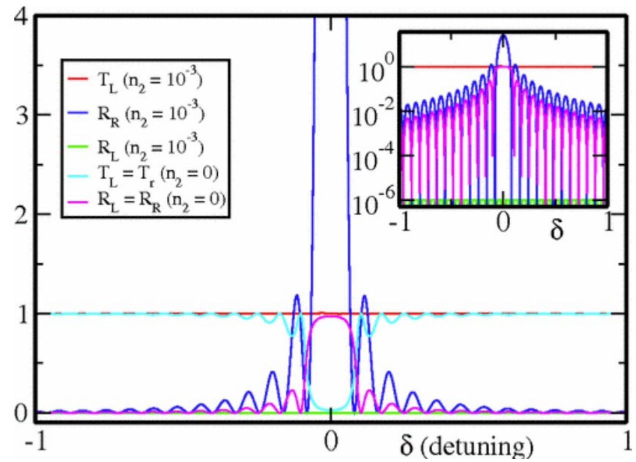
$$\langle\chi| \neq |\psi\rangle^{\dagger}$$

- Exceptional point (EP): → Eigenvectors coalesce at EP and do not form complete set.

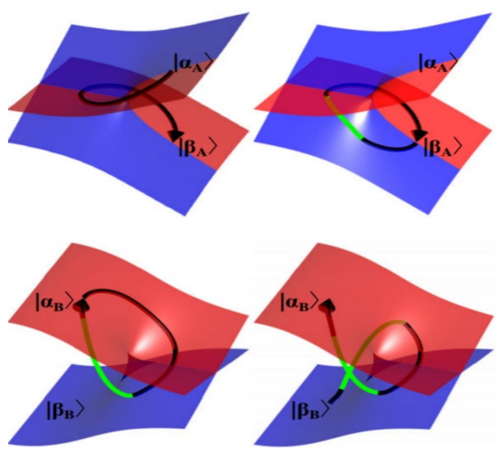
Exceptional Points



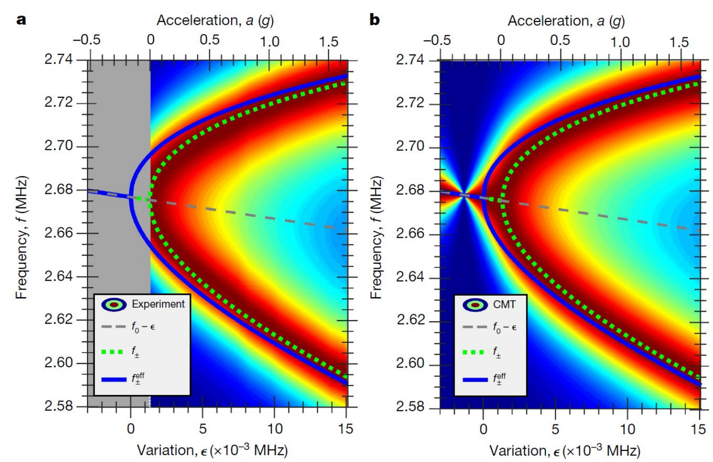
Critical phenomena: L. Xiao et, al., Phys. Rev. Lett. 123, 230401 (2019).



Unidirectional invisibility: Z. Linet, al., Phys. Rev. Lett. 106, 213901 (2011).

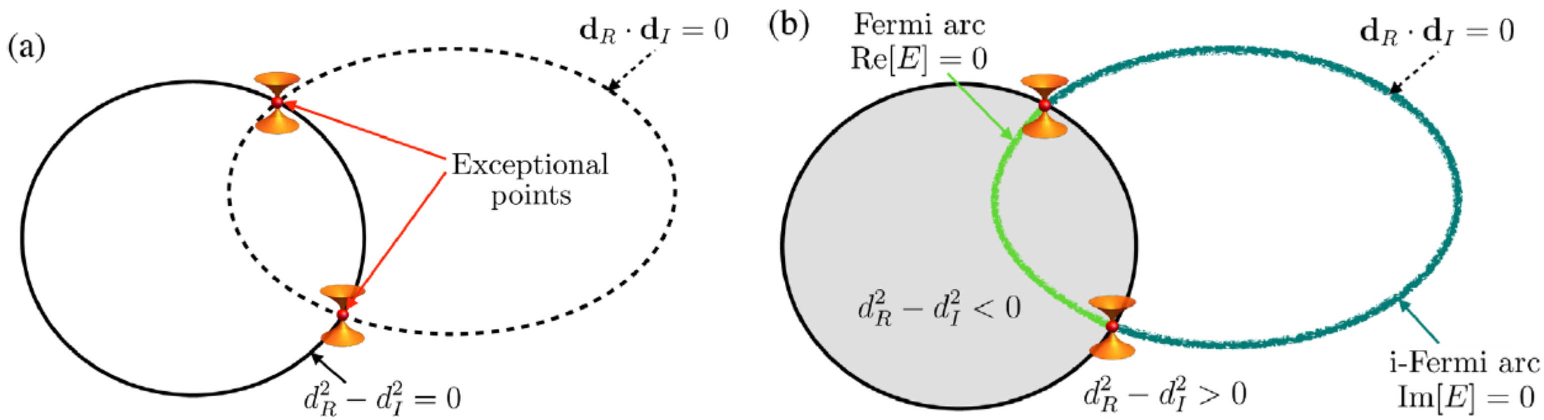


Mode switching: W. Liu et, al., Phys. Rev. Lett. 126, 170506 (2021) .



Sensor: R. Kononchuk et, al., Nature 607, 697–702 (2022).

Exceptional Points



Emil J. Bergholtz, et al. Rev. Mod. Phys. 93.015005 (2021).

How to construct EPs?

$$H(k) = \mathbf{d}(k) \cdot \boldsymbol{\sigma} + d_0(k)\boldsymbol{\sigma}_0, \ \mathbf{d} \equiv \mathbf{d}_R + i\,\mathbf{d}_I$$

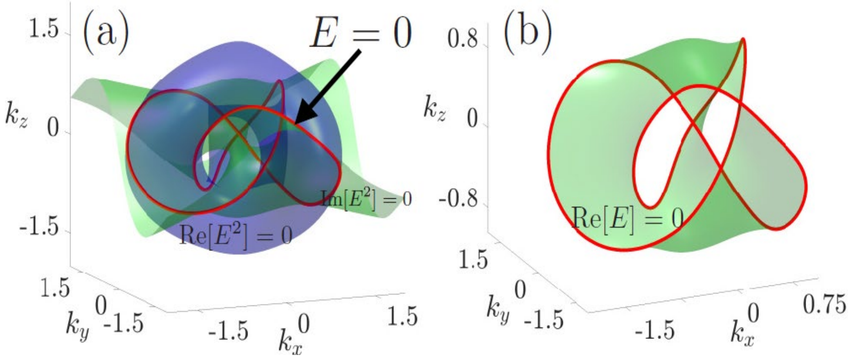
$$E_\pm = d_0 \pm \sqrt{\mathbf{d}_R^2 - \mathbf{d}_I^2 + 2\mathbf{d}_R\mathbf{d}_I}$$

Two EP conditions: $\mathbf{d}_R^2 - \mathbf{d}_I^2 = 0$, $\mathbf{d}_R \cdot \mathbf{d}_I = 0$. EPs generic exist in 2D systems.

Exceptional Points

Without Symmetry

- EPs form closed nodal lines in 3D.
- New form of topological phases that is stable against perturbations.



Johan Carlström, et al. Phys. Rev. B 99, 161115(R) (2019).

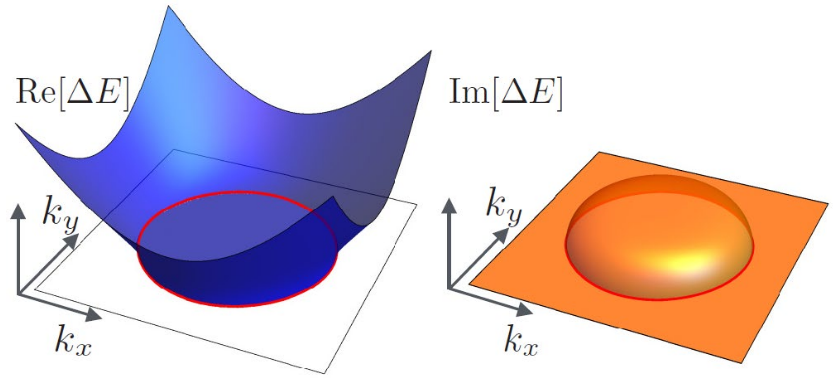
With Symmetry

For example: Q: $H = qH^{\dagger}q^{-1}$,

$$q^{\dagger}q^{-1} = qq^{\dagger} = \mathbf{I}.$$

Symmetry Q trivially implies EP condition $\mathbf{d}_R \cdot \mathbf{d}_I = 0$.

→ Lines of symmetry protected EPs in 2D, isolated EPs in 1D.



Jan Carl Budich, et al. Phys. Rev. B 99, 041406(R) (2019).

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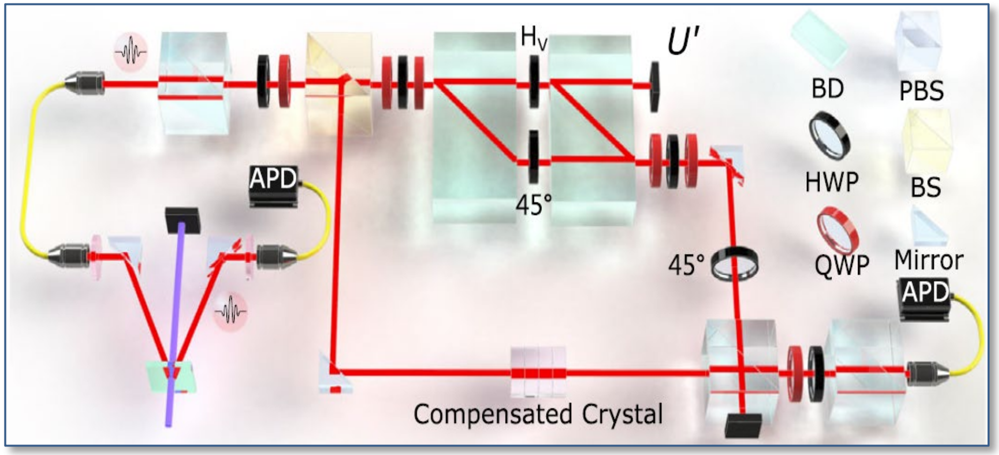
Exceptional Non-Hermitian Metals

Mapping:

$$H'(\mathbf{k}) = H(\mathbf{k}) + d_0\sigma_0,$$

Evolution:

$$U' = R_2 L(\theta_V) R_1$$



Process:

$$\{|H\rangle = (1,0)^{\mathrm{T}}, |V\rangle = (0,1)^{\mathrm{T}}\},$$

$$|\Psi_j\rangle = \frac{1}{\sqrt{2}} |\psi_j\rangle \left(|t\rangle + |r\rangle\right), j = \pm,$$

$$|\Psi'_j\rangle = \frac{1}{\sqrt{2}} \left(e^{-iE'_j} |\psi_j\rangle |t\rangle + |\psi_j\rangle |r\rangle\right).$$

Interferometric measurements:

$$N_H = \mathcal{N}\frac{|\alpha_j|^2}{2}, N_V = \mathcal{N}\frac{|\alpha_j|^2}{2} e^{2\mathrm{Im}(E'_j)},$$

$$N_+ = N_H \left(\frac{1}{2}e^{iE'_j} + \frac{1}{2}e^{-iE'_j} + \frac{1}{2} + \frac{1}{2}e^{2\mathrm{Im}(E'_j)}\right)$$

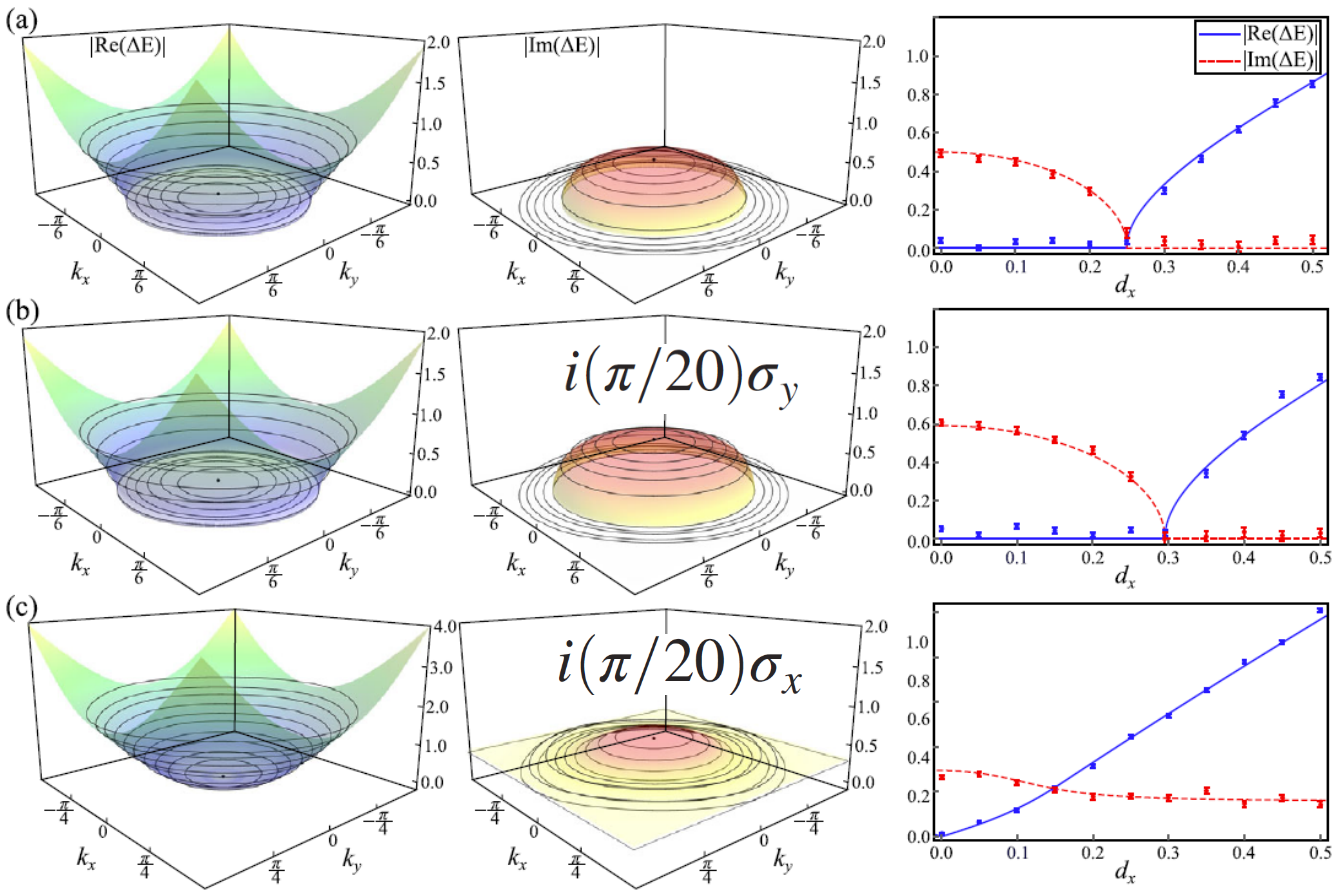
$$N_R = N_H \left(\frac{i}{2}e^{-iE'_j} - \frac{i}{2}e^{iE'_j} + \frac{1}{2} + \frac{1}{2}e^{2\mathrm{Im}(E'_j)}\right)$$

Kunkun Wang, Lei Xiao, Jan Carl Budich*, Wei Yi*, and Peng Xue*, Physical Review Letters:127.026404 (2021);

Exceptional Non-Hermitian Metals

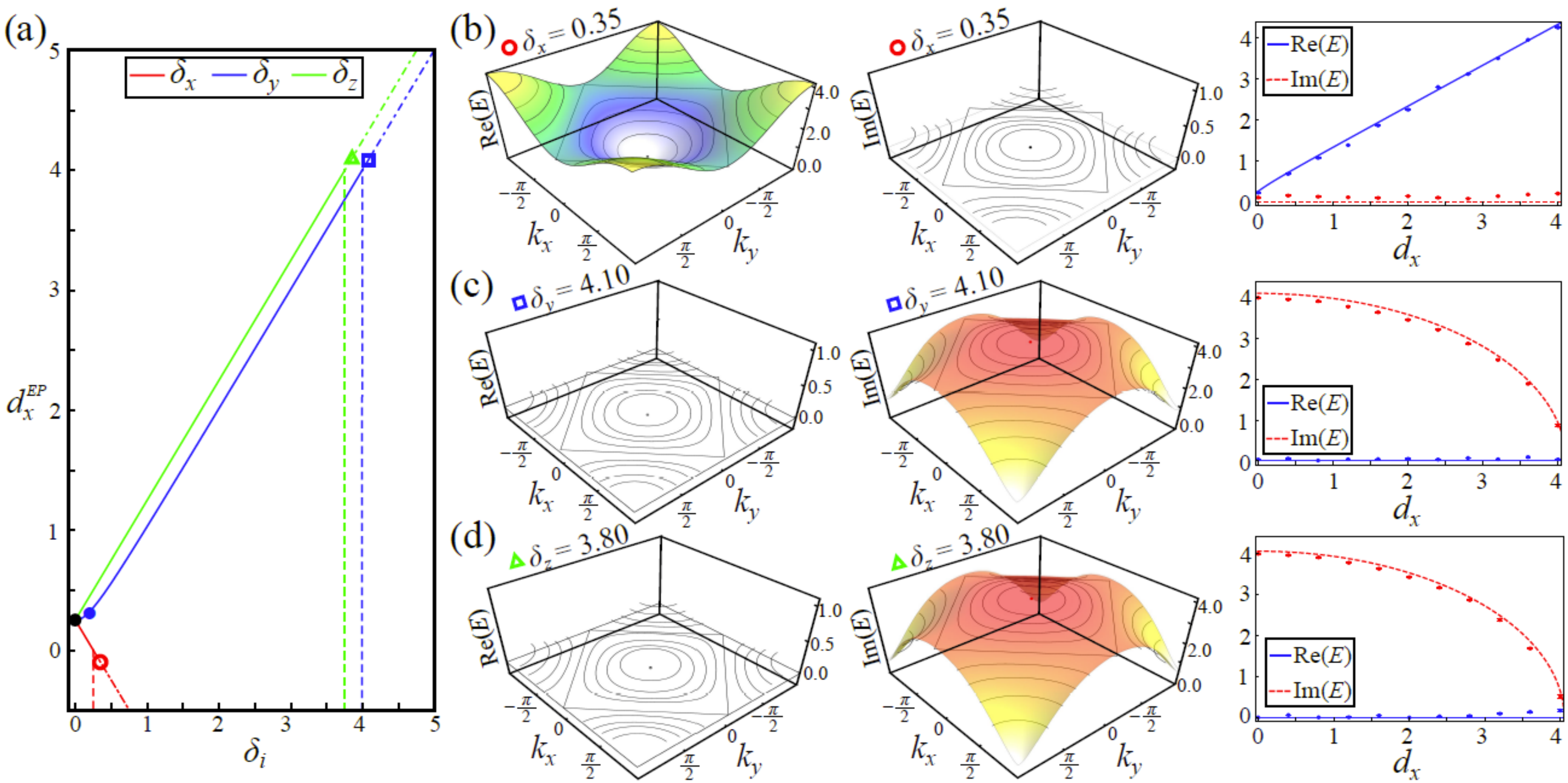
2D NH Hamiltonian with symmetry-protected EIs:

$$H_1 = (2 - \cos k_x - \cos k_y)\sigma_x + \frac{i}{4}\sigma_z. \quad \text{with} \quad d_x = 2 - \cos k_x - \cos k_y,$$



Exceptional Non-Hermitian Metals

2D NH Hamiltonian with symmetry-protected EIs under perturbation:



Exceptional Non-Hermitian Metals

Symmetry-independent knotted EIs:

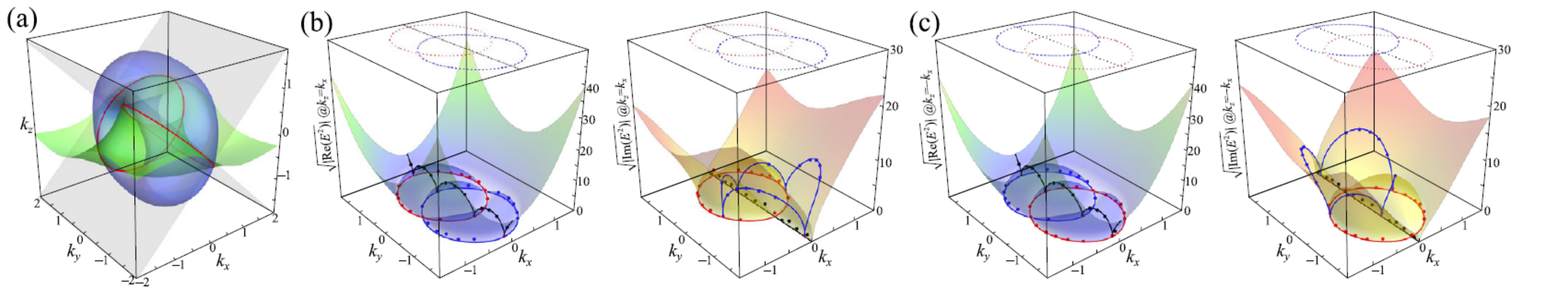
$$\mathbf{d}_R(\mathbf{k}) = [f_1(\mathbf{k}) - \epsilon, \epsilon, 0], \quad \mathbf{d}_I(\mathbf{k}) = [0, f_2(\mathbf{k}), \sqrt{2}\epsilon] \quad f_1(\mathbf{k}) + i f_2(\mathbf{k}) = Z_0^p + Z_1^q,$$

Choose:

$$Z_0 = \sin k_x + i \sin k_y,$$

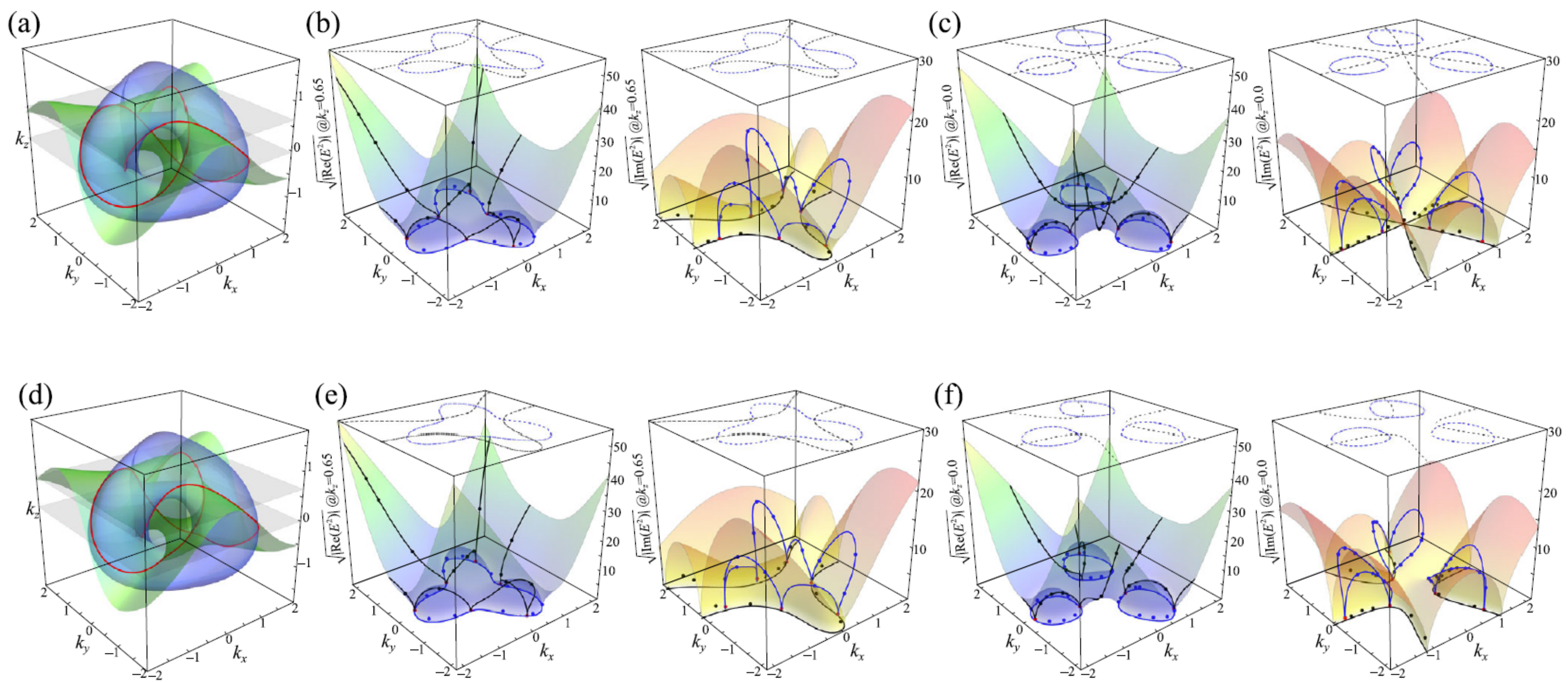
$$Z_1 = 2 \sum_{\alpha = x,y,z} \cos k_\alpha - 5 + i \sin k_z$$

Hopf linked EIs (*p*=2, *q*=2) :



Exceptional Non-Hermitian Metals

Trefoil knotted EL (p=3, q=2):



Effects of perturbations $\sum_{i=x,y,z} \delta_i \sigma_i$, where δ_i and σ_i $\in$ [0, 0.4] are chosen randomly.

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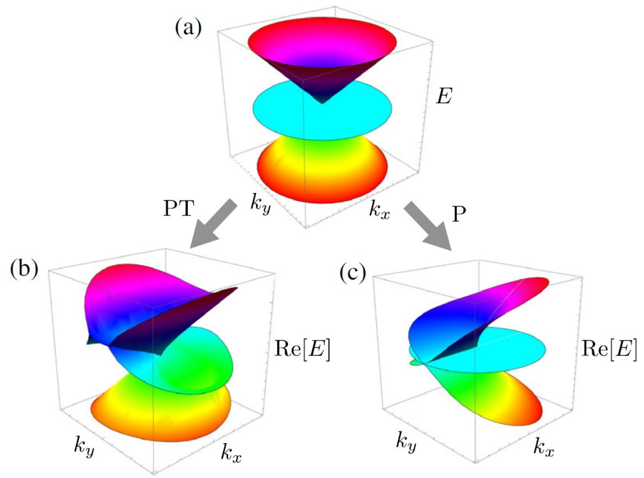
Higher-Order Exceptional Points

Higher-Order EPs

- *n*th-order EP is stable in a (2*n*-2)-dimensional NH system.
- The perturbation of ε can give rise to an energy shift $\sim\varepsilon^{1/n}$.
- Generic NH symmetries can reduce the dimension for the occurrence of the third-order EPs.
- Different symmetries also entail qualitatively different phenomenology for EPs with the same order.

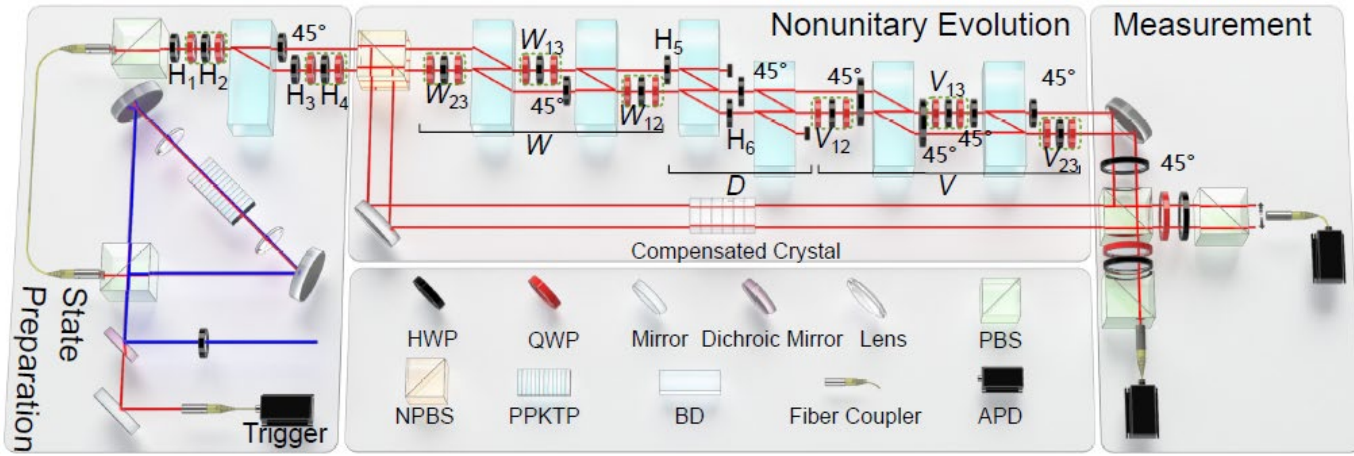
Exceptional nodal topologies		
Dimension	Without $\mathcal{PT}$	With $\mathcal{PT}$
$D=1$	—	2EPs
$D=2$	2EPs	Lines of 2EPs 3EPs
$D=3$	Knots of 2EPs	Surfaces of 2EPs Knots of 3EPs 4EPs

M. Stålhammar et al., Phys. Rev. B 104, L201104, (2021).

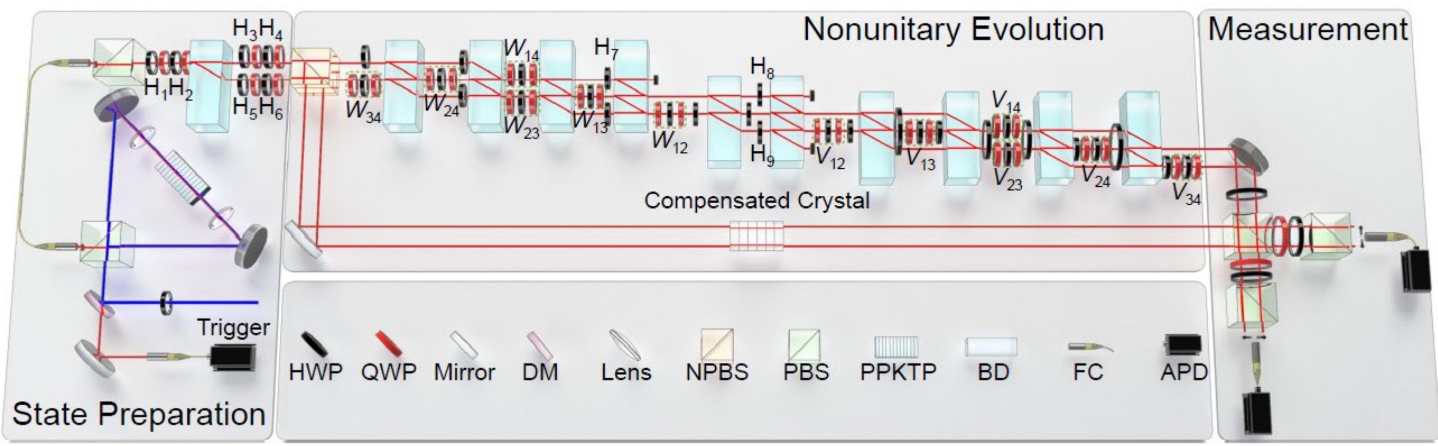


I. Mandal et, al. Phys. Rev. Lett. 127, 186601 (2021).

Three-band model:



Four-band model:



Arbitrary multi-band model: …

Kunkun Wang, Lei Xiao, Haiqing Lin, Wei Yi*, Emil J. Bergholtz*, and Peng Xue*, arXiv:2303.11834 (2023);

Higher-Order Exceptional Points

Higher-spin Dirac-like NH Hamiltonians with PT symmetry in a 2D reciprocal:

$$H_{PT} = k_x\lambda_1 + i\epsilon(\lambda_2+\lambda_4+\lambda_5) + (k_y - i\epsilon)\lambda_6 - (k_y+i\epsilon)\lambda_7 - \frac{\epsilon(\lambda_3+\sqrt{3}\lambda_8)}{2}.$$

Here λ_i denotes the 3×3 Gell-Mann matrix.

- PT symmetry:

$$(PT)H_{PT}^{*}(PT)^{-1} = H_{PT}$$

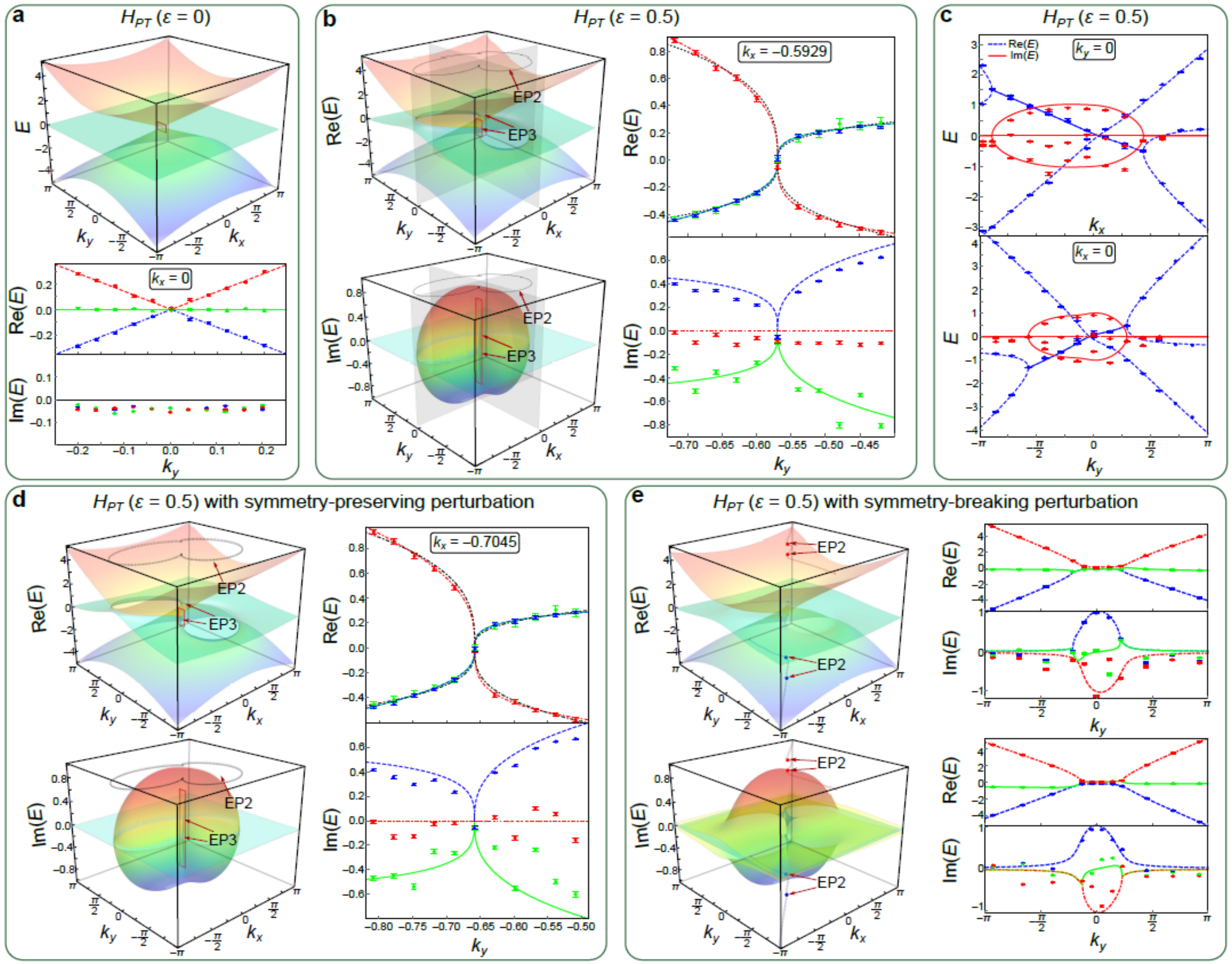
P = diag(1, -1, 1) and *T* = diag(-1, 1, i).

- Symmetry-preserving perturbation:

$$i\frac{\pi}{20}(\lambda_4+\lambda_5)$$

- Symmetry-breaking perturbation:

$$\sum_{i=1}^{8}\delta_i\lambda_i$$



Higher-Order Exceptional Points

NH Lieb lattice with P symmetry in a 2D reciprocal:

$$H_P = (1+\cos k_x - i\epsilon)\lambda_1 + (1+\cos k_y + i\epsilon)\lambda_6 - \sin k_x \lambda_2 - \sin k_y \lambda_7,$$

Here λ_i denotes the 3×3 Gell-Mann matrix.

- P symmetry:

$$H_P = -PH_PP^{-1}$$

P = diag(1, -1, 1).

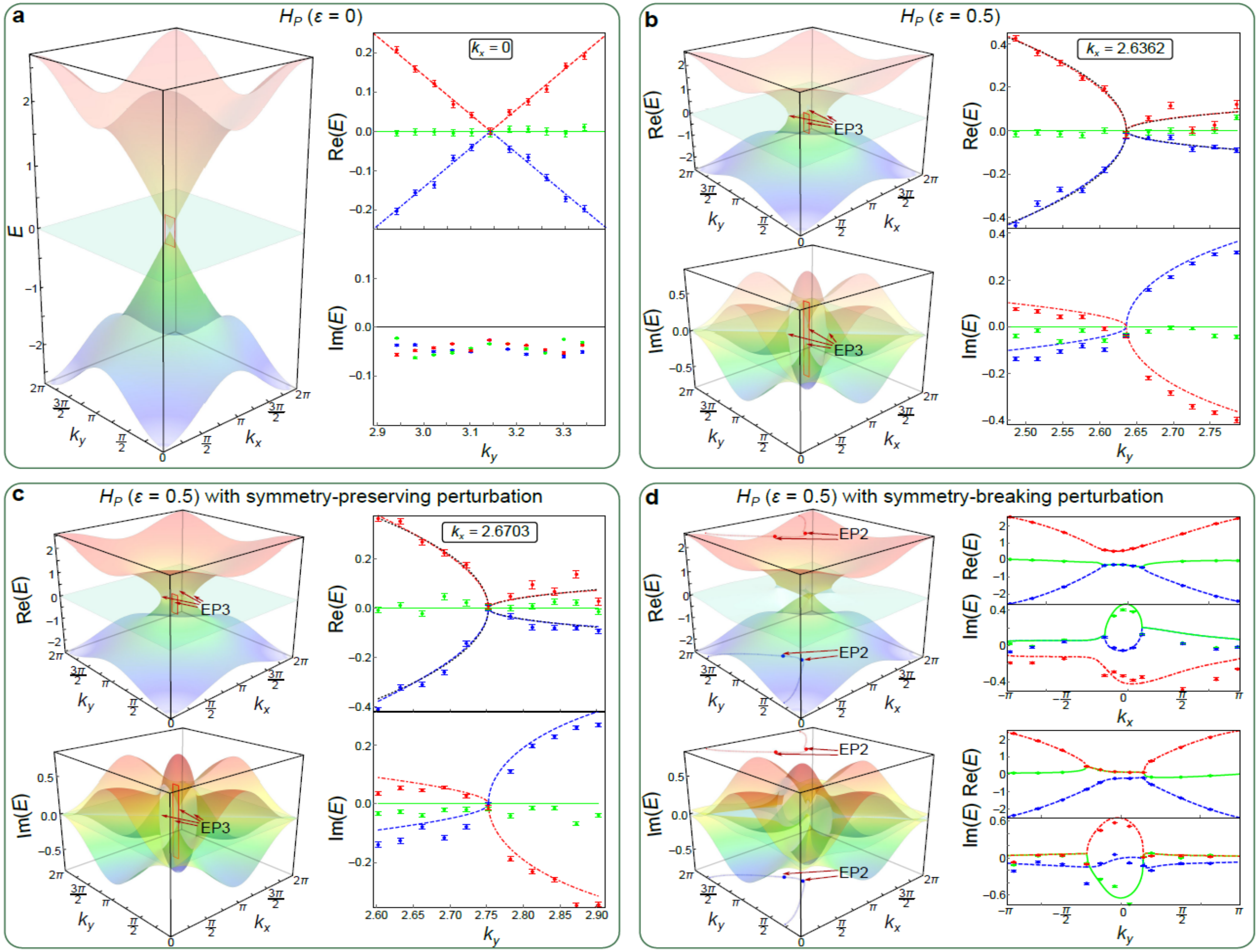
- Symmetry-preserving perturbation:

$$i\frac{\pi}{20}\lambda_1$$

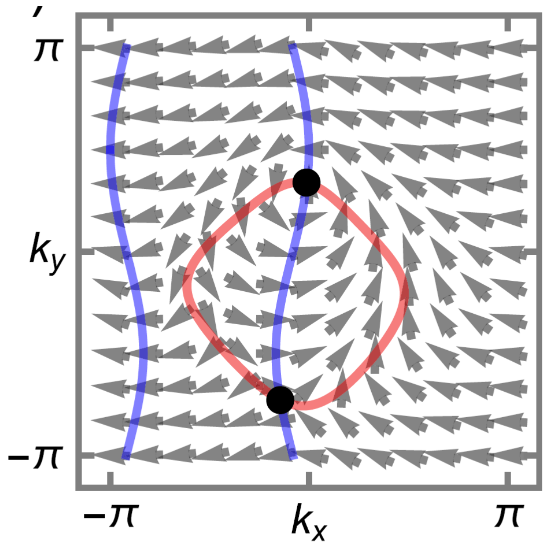
- Symmetry-breaking perturbation:

$$\sum_{i=1}^{8} \delta_i \lambda_i$$

$$\delta_i \in \left[-\frac{\pi}{20}, \frac{\pi}{20}\right]$$

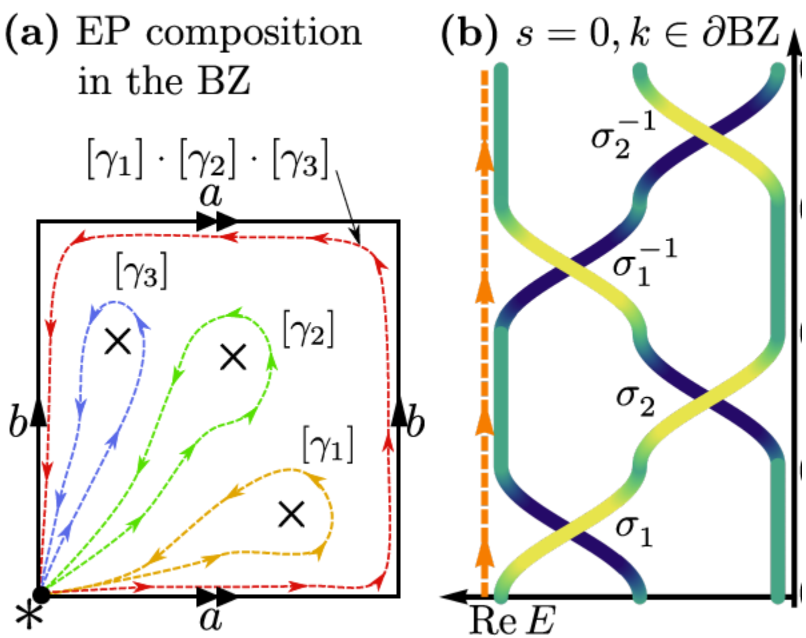


Monopole

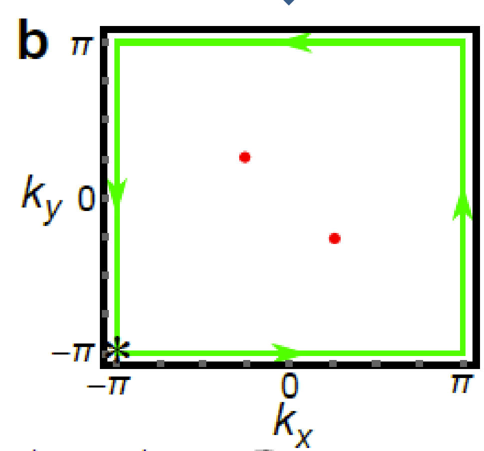


Monopoles of band-structure degeneracies are subject to well-established no-go (doubling) theorems: **exceptional points come in pairs.**

Z. Yang et, al., Phys. Rev. Lett. 126, 086401 (2021).



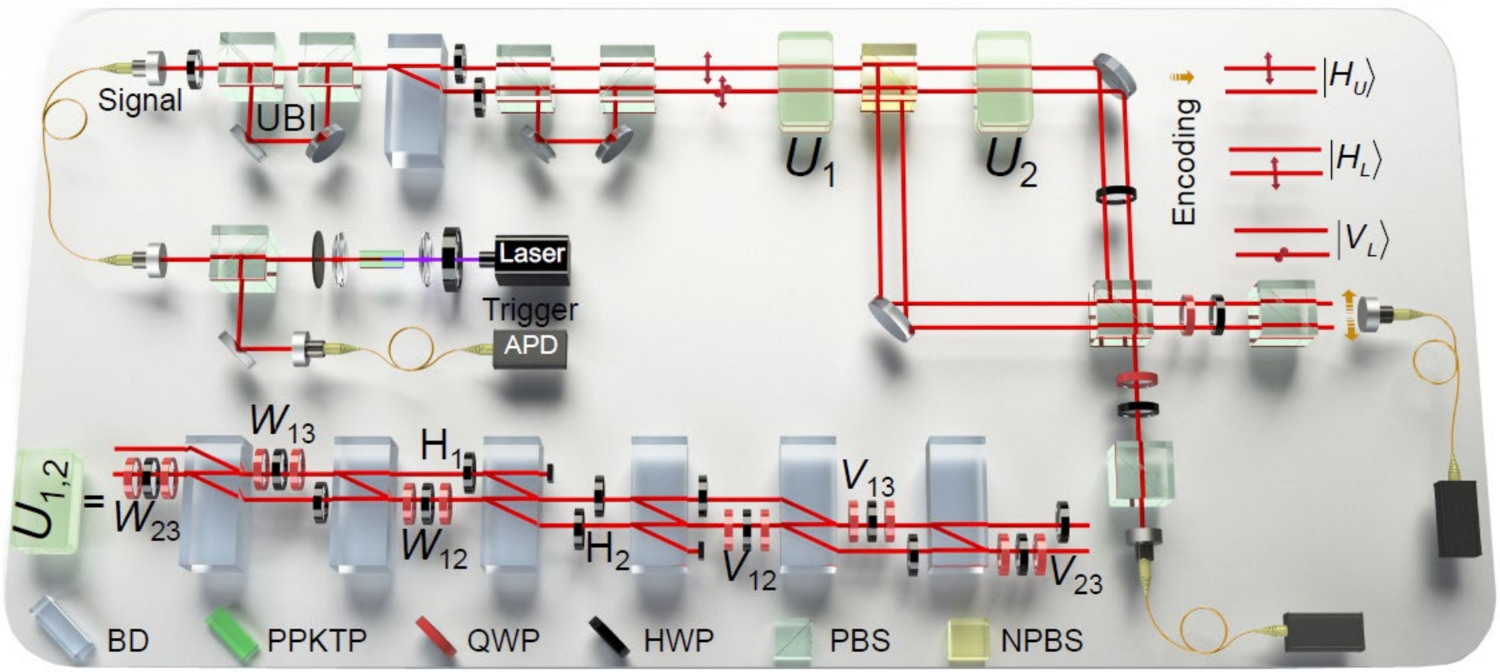
$$\sigma_{j+1}\sigma_j\sigma_{j+1} = \sigma_j\sigma_{j+1}\sigma_j, \;\; \sigma_i\sigma_j = \sigma_j\sigma_i \; (|i-j| > 1).$$



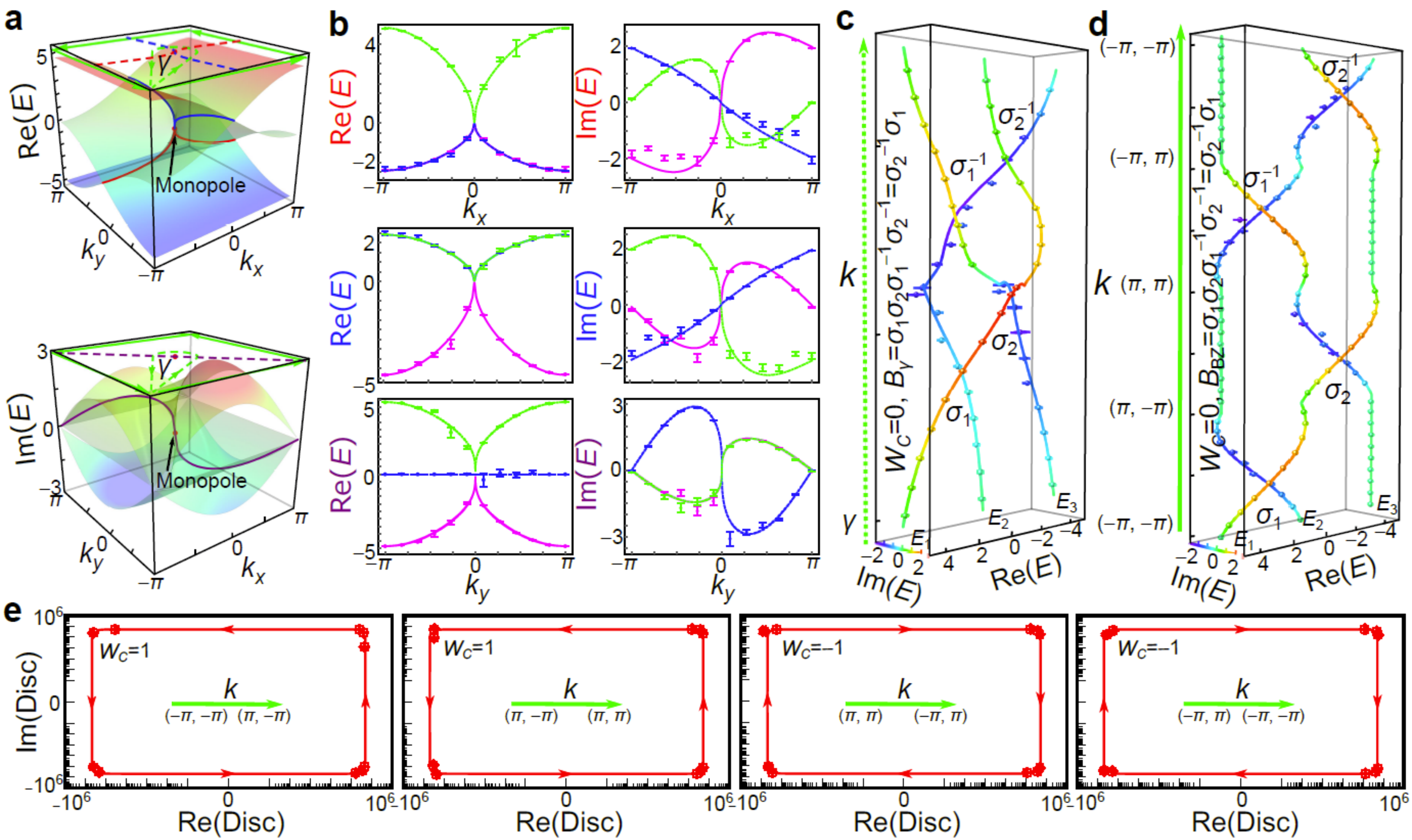
Kunkun Wang, J. Lukas K. Konig, Kang Yang, Lei Xiao, Wei Yi, Emil J. Bergholtz*, and Peng Xue*, arXiv: 2410. 08191 (2024).

Photonic Non-Abelian Braid Monopole

$$H_M^\delta = \begin{pmatrix} \delta - e^{ik_x} & -i(1+e^{ik_x}) & 0 \\ -i(1+e^{ik_x}) & e^{ik_x} - e^{ik_y} & -i(1+e^{ik_y}) \\ 0 & -i(1+e^{ik_y}) & -\delta + e^{ik_y} \end{pmatrix}. \quad (1)$$



Photonic Non-Abelian Braid Monopole



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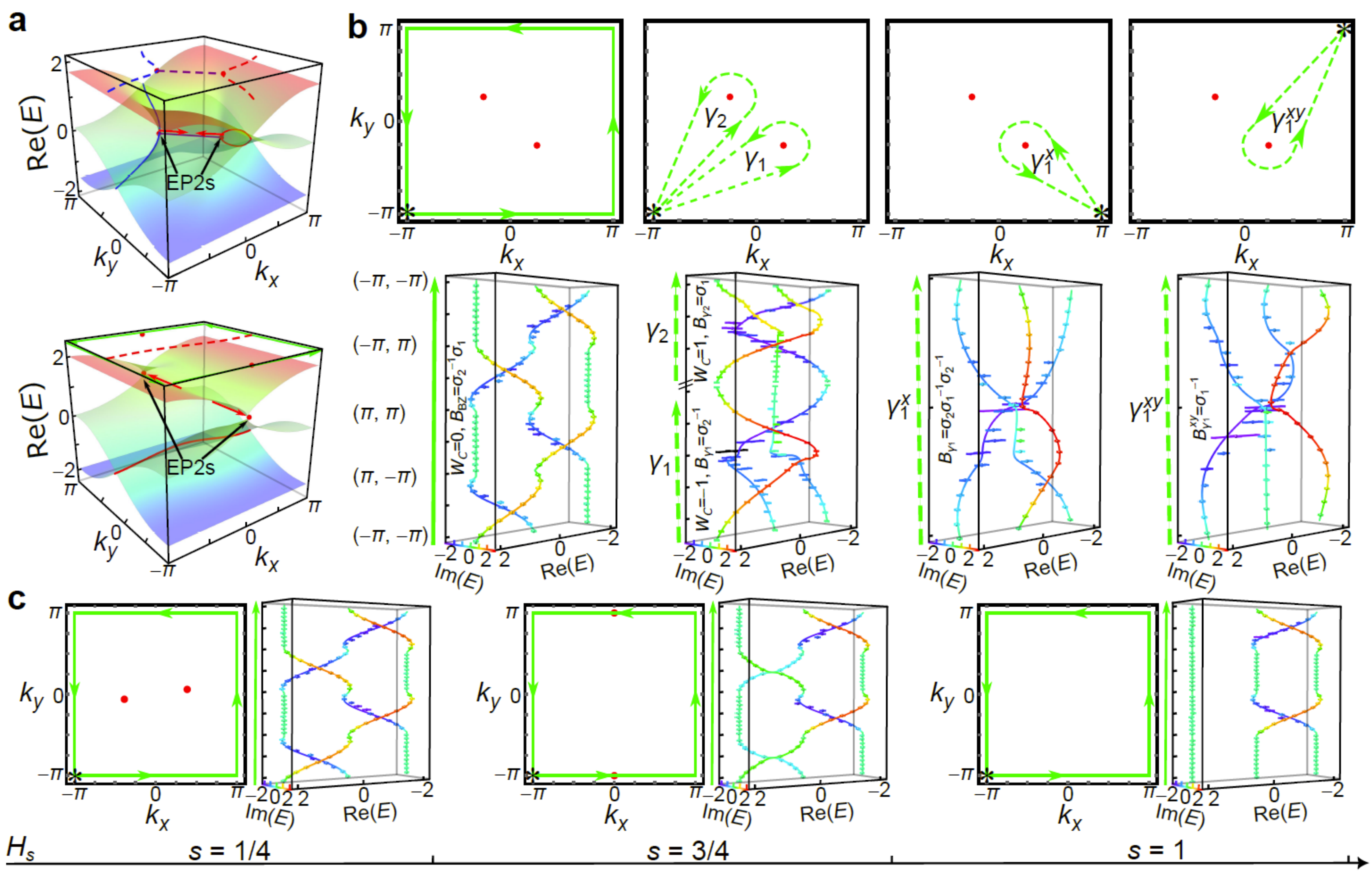

Summary

- We experimentally simulate two distinct types of NH metals:
  - ✓ Two-dimensional systems with symmetry-protected Els.
  - ✓ Three-dimensional systems possessing symmetryi ndependent topological ELs in the form of knots.

- We experimentally observe two different symmetry-protected third-order EPs.
  - ✓ The third-order EP exhibits a generic $\sim k^{1/3}$ dispersion enforced by PT symmetry.
  - ✓ An anomalous $\sim k^{1/2}$ dispersion for the chiral-P-symmetry-protected third-order EP.

- We experimentally presents the first experimental realization of a **non-Abelian monopole degeneracy** in a non-Hermitian three-band system, embodied as a third-order exceptional point.

Cooperators

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(University of Science and Technology of China)

Haiqing Lin

(Zhejiang University)

Jan Carl Budich

(Technische Universität Dresden)

Emil J. Bergholtz

(Stockholm University)