Causal decompositions of 1D QCAs

Augustin Vanrietvelde QSQW Conference January 16th, 2025









Looking for a constructive form of QCAs

1D Quantum Cellular Automata



What are the possible 1D QCAs?



No clear view of how 1D QCAs 'look like' in general

Most satisfactory result would be a **constructive form**: 'QCAs are all and only the unitaries constructible in a certain way'

A constructive form of 1D QCAs for $r = \frac{1}{2}$



(Also yields index theory)

Higher radii?



Usual story: "just regroup"... ...but that gives a decomposition of radius 3/2

At radius > $\frac{1}{2}$, we don't have a constructive form of 1D QCAs yet.

Causal decompositions

Causal decompositions: one-way causation



Eggeling, Schlingemann and Werner, "Semicausal operations are semilocalizable", 2001 (quant-ph/0104027)

Causal decompositions: the diamond



Lorenz and Barrett, "Causal and compositional structure of unitary transformations", 2020 (2001.07774)

Causal decompositions: the general conjecture



Quantum Information-ish

12

Causal decs yield constructive forms



Precisely a causal decomposition!



A theory of partitions of quantum systems – a surprisingly subtle problem

A bit of mereology



A bit of <u>quantum</u> mereology



A stumbling block: non-factor C* algebras

Wedderburn-Artin theorem

Any fin-dim C* algebra satisfies

 $\omega \cong \bigoplus_{k \in K} \operatorname{Lin}\left(\mathcal{H}_{\Omega}^{k}\right)$



A C*-algebra of blockdiagonal matrices



Incorporating non factorness



accounting for non-factorness is key!

Leads to numerous complications due to Failure of Local Tomography (FOLT) Has numerous (and more physically motivated) other potential applications

A theory of partitions (summed up)

Goal: defining partitions into \geq 3 parts



Non-factorness and FOLT lead to a surprising wealth of ambiguities



It has a lot of deep structure to explore!!



We overcome that, and provide a well-defined and consistent theory



Causal decompositions of 1D QCAs

Our result: all 1D QCAs are causally decomposable



Our result: all 1D QCAs are causally decomposable



1D QCAs are causally decomposable



Implications

- All causal dynamics in (discretised) 1+1 spacetime are (<u>unitarily</u>) localisable

- **Constructive form** of all 1D QCAs \rightarrow Extremely tight handle on them

Some lessons from this talk

We need constructive forms for QCAs

Causal decompositions yield constructive forms

We would be wise to pay more attention

to non-factor C* algebras

In 1+1D discretised spacetime, the phenomenological and

ontological formulations of relativistic causality are equivalent

Prospects



