Effective dynamics from minimising dissipation

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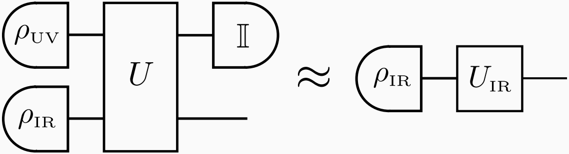
Question

Effective unitary theory for the coarse grained dynamics of a quantum system evolving in discrete time steps?



Efficient description of physics at a given scale. Rough idea: *effect of high-energy dofs can be encapsulated in a few parameters of an effective low-energy unitary theory.*

Well-established rules for determining this effective theory for Hamiltonian dynamics.

However, system evolving in discrete time: largely unexplored[1].

[1] But not completely: S. Boettcher, S. Falkner, and R. Portugal. In: *Physical Review A* 90.3 (2014); P. Arrighi, S. Facchini, and M. Forets. In: *New Journal of Physics* 16.9 (2014); O. Duranthon and G. Di Molfetta. In: *Physical Review A* 103.3 (2021); L. S. Trezzini, A. Bisio, and P. Perinotti. In: *arXiv:2407.12652* (2024)

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Quantum walks (QW) and quantum cellular automata (QCA):

- Universal quantum computer[2] and quantum simulation;
- QW: search algorithms[3];
- QCA: classification of Floquet systems[4].

Trotterised Hamiltonian dynamics.

[2] Childs, Physical review letters 102 (2009); Watrous, Proc. IEEE 36th ann. foundations of CS (1995).
[3] Aharonov et al., Proc. 33rd annual ACM symposium on Theory of computing (2001).
[4] Po et al., Physical Review X 6 (2016).

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Using a unitary approximation instead of the exact non-unitary channel simplifies both the analytical study and the simulation of the dynamics.

More fundamental level: to understand which models of discrete dynamics could be viable UV completion of known unitary quantum field theories.

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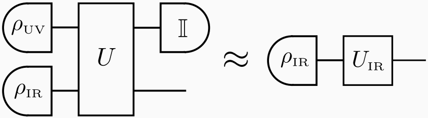
Goal: find U_{IR} such that



How: maximise channel fidelity[5].

[5] Raginsky, Physics Letters A 290 (2001).

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Treat ρ_{IR} as an open quantum system coupled to a bath of UV degrees of freedom.

Maximising the channel fidelity corresponds to minimise dissipation[6] to this bath.

[6] Hayden and Sorce, Journal of Physics A: Mathematical and Theoretical 55 (2022); Colla and Breuer, Physical Review A 105 (2022).

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Weak coupling

Why: we cannot hope to find a good unitary effective dynamics in general.

We consider unitaries

$$U = (V_{\mathrm{IR}} \otimes V_{\mathrm{UV}})U_{\mathrm{MIX}}(\theta), \quad U_{\mathrm{MIX}} = \mathbb{1} + i\theta H_{\mathrm{MIX}} + O(\theta^2).$$

For $\theta \ll 1$ they approximately factorize and the IR dynamics should be well-approximated by a unitary effective dynamics.

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Main result

Assume: $U = (V_{\text{IR}} \otimes V_{\text{UV}})U_{\text{MIX}}(\theta)$ with $U_{\text{MIX}} = \mathbb{1} + i\theta H_{\text{MIX}} + O(\theta^2)$

Effective theory is mean-field average

$$U_{\text{IR}} = V_{\text{IR}} e^{i\theta H_{\text{IR}}}, \quad H_{\text{IR}} = \text{Tr}_{\text{UV}}[(\rho_{\text{UV}} \otimes \mathbb{1}_{\text{IR}})H_{\text{MIX}}].$$

Error is a sum of energy variances

$$1 - \mathcal{F} = \theta^2 \sum_{\lambda \neq 0} \left[\text{Tr}\left(\rho_{\text{UV}} H^2_{\text{UV},\lambda}\right) - \left(\text{Tr}\, \rho_{\text{UV}} H_{\text{UV},\lambda}\right)^2 \right],$$

where $H_{\text{MIX}} = \sum_\lambda H_{\text{UV},\lambda} \otimes H_{\text{IR},\lambda}$.[7]

[7] Given a basis $\{H_{\text{IR},\lambda}\}$ such that $H_{\text{IR},0} = \mathbb{1}_{\text{IR}}$, and $\text{Tr}\left[H_{\text{IR},\lambda} H_{\text{IR},\lambda'}\right] = d_{\text{IR}} \delta_{\lambda,\lambda'}$.

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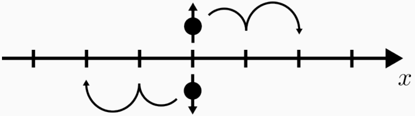
QW on $\mathbb{Z}$ with 2-dim internal dof, generated by

$$U = \begin{pmatrix} \cos(\theta)\, T^{\dagger} & -i\sin(\theta) \\ -i\sin(\theta) & \cos(\theta)\, T \end{pmatrix}, \quad T\,|x\rangle = |x-1\rangle\,.$$

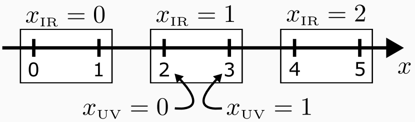


Reproduces the Dirac equation in the limit of small momenta[8].

[8] Bisio, D'Ariano, and Tosini, Annals of Physics 354 (2015).

Coarse-graining

We group lattice sites in pairs and relabel positions as $|x\rangle \to |x_{\mathrm{IR}}\rangle \otimes |x_{\mathrm{UV}}\rangle$, with $x = 2x_{\mathrm{IR}} + x_{\mathrm{UV}}$



Time coarse-graining: find effective theory for U^2. This way the light cone is preserved.

Result

The effective dynamics is[9]

$$U_{\text{IR}} = \begin{pmatrix} \cos(\theta_{\text{IR}})\, T_{\text{IR}}^{\dagger} & -i\sin(\theta_{\text{IR}}) \\ -i\sin(\theta_{\text{IR}}) & \cos(\theta_{\text{IR}})\, T_{\text{IR}} \end{pmatrix}, \quad \theta_{\text{IR}} = 2\,\text{Tr}[\rho_{\text{UV}}\sigma_X]\theta.$$

Depending on ρ_{UV} the approximation error is $\theta^2/2 \leq 1 - \mathcal{F} \leq 2\theta^2$.

[9] In the limit of small momenta.

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Summary

- Effective dynamics by *minimizing dissipation*;
- Weak coupling: effective dynamics is a mean-field average over the UV dof, and the error is a sum of energy variance;
- Dirac QW: the effective dynamics is obtained by a rescaling of θ.

Future directions

- Consider other QW, in particular interacting ones;
- Consider Hilbert spaces that only approximately factorise.

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Conclusions

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Thank you!

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