Quantum state estimation of optical states via quantum extreme learning machines

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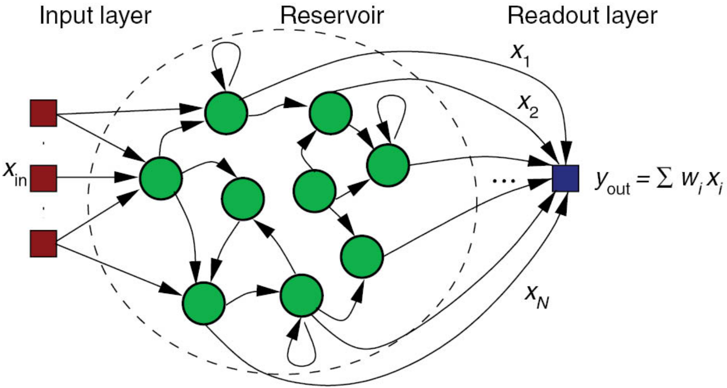
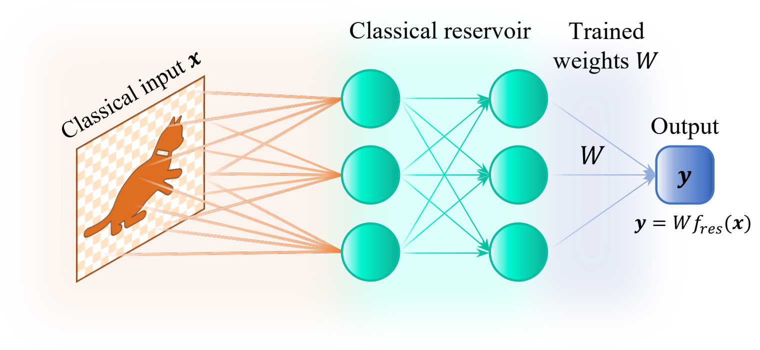
Classical reservoir computing (RC)



Machine learning protocol that allows to process time series thanks to the memory properties of the “reservoir”.

Classical *extreme learning machines* (ELMs)

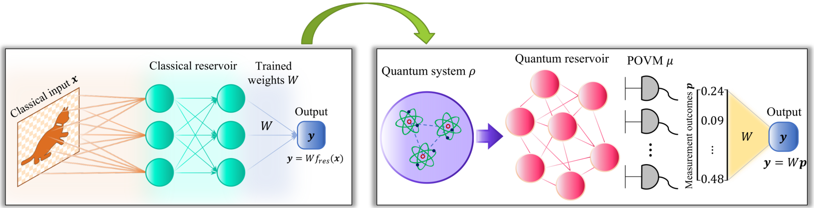


ELMs are the memoryless counterparts of RCs.

Classical *extreme learning machines* (ELMs)

- **Goal**: find a *linear* map W such that $W\mathbf{x}_i \simeq \mathbf{f}(\mathbf{x}_i)$ for some *target behaviour* $\mathbf{f}$.
- **How?** Using a supervised learning training method, *i.e.* the training is a function $\{(\mathbf{x}_i, \mathbf{y}_i)\}_{i=1}^{N_{\rm train}} \to W$ for a given training dataset $\{(\mathbf{x}_i, \mathbf{y}_i)\}_{i=1}^{N_{\rm train}}$.
- **Why?** The linearity of the model makes the training "extremely" fast, at the cost of reduced expressivity.

From classical to quantum ELMs

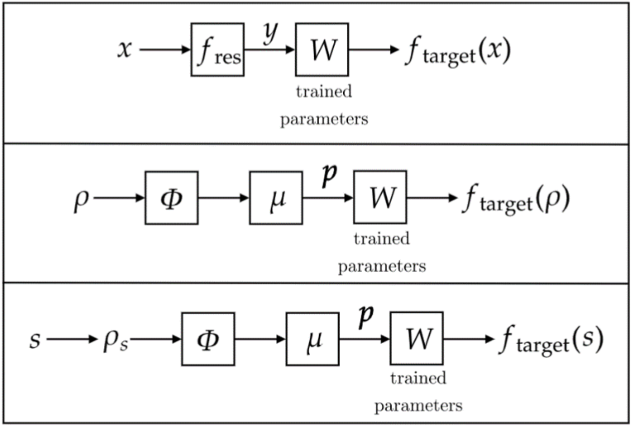


- Inputs are states ρ_i; the reservoir dynamic is a *channel* Φ; the measurement is a *POVM* $\boldsymbol{\mu}$.
- The final function is computed as a linear combination of measurement probabilities:

$$W\mathbf{p}(\rho) = \sum_b W_b \operatorname{tr}(\mu_b \Phi(\rho)) \in \mathbb{R}. \tag{1}$$

Classical VS *quantum* information processing

One can process classical or quantum information, depending on the problem setting:



General statements about reachability

- Linearity of quantum channels and measurements allow an easy characterisation of achievable target functions:

$$\sum_j W_j p_j = \sum_j W_j \operatorname{tr}(\mu_j \Phi(\rho)). \tag{2}$$

Defining an "effective measurement" $\tilde{\mu}_j \equiv \Phi^\dagger(\mu_j)$, the set of all (exactly) achievable observables is precisely

$$\operatorname{span}_{\mathbb{R}}(\{\tilde{\mu}_j\}). \tag{3}$$

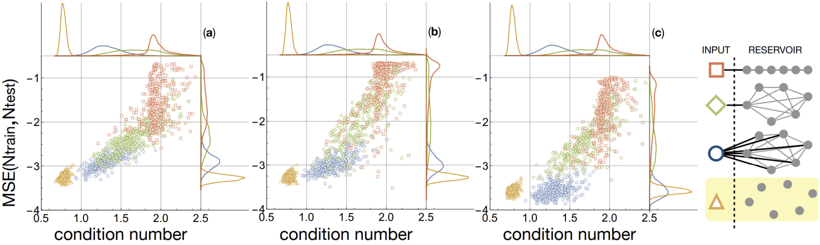
- For example, you can't use a QELM to estimate accurately the concurrence of input states, but you can reconstruct entanglement witnesses.

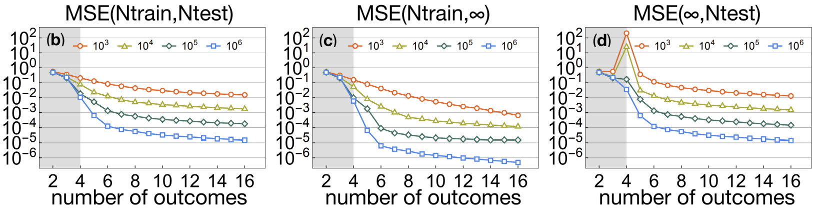
General statements about reachability

- To reconstruct arbitrary observables of (d-dimensional) input states, you need $\geq d^2$ outcomes. Though having just d^2 outcomes is often numerically unstable.
- For "random" reservoir dynamics, the more outcomes the better (but *too* many don't really help).

So... why QELMs? No need to "fine tune" the dynamics; easy training; no overfitting; flexible architecture. Makes for an experimental robust platform.

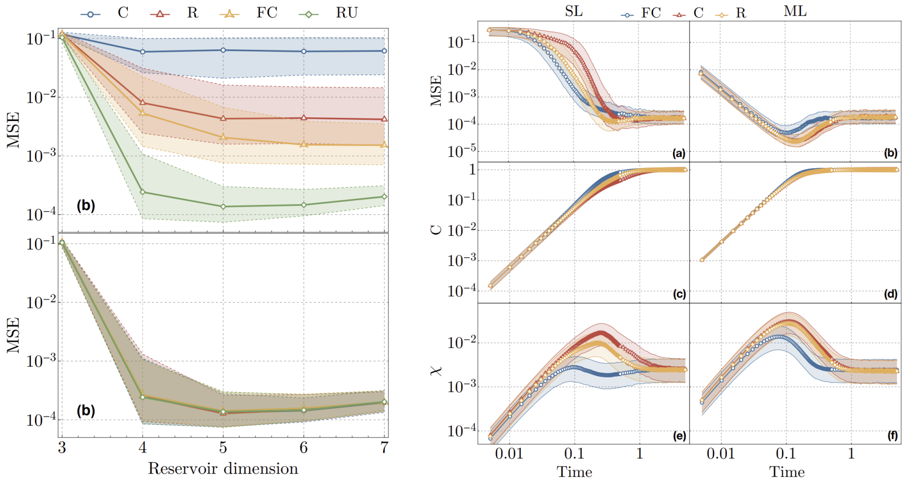
Typical performances of QELMs





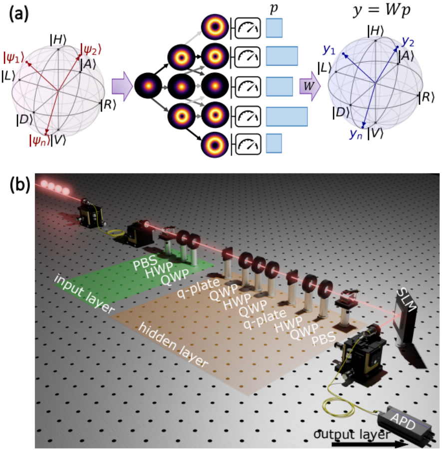
Innocenti et al., Communications Physics 6.1 (2023): 118

Results: QELMs work well beyond the scrambling time



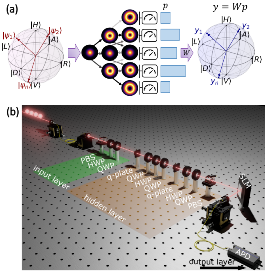
Marco Vetrano et al. arXiv:2409.06782 (accepted on npj:qi)

Photonic experimental implementation



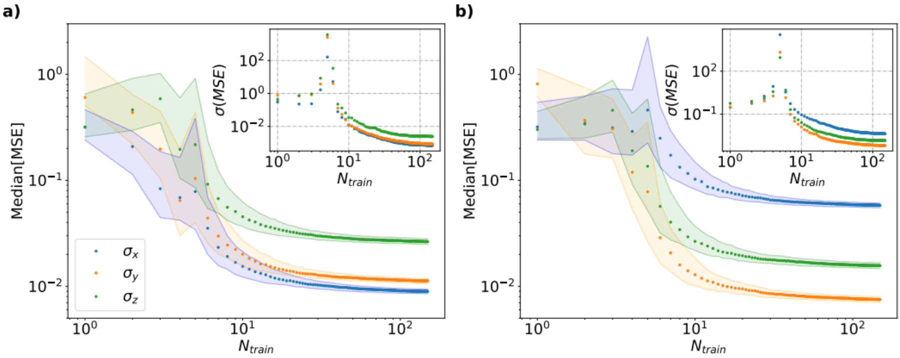
Alessia Suprano et al. Physical Review Letters 132.16 (2024): 160802.

Photonic experimental implementation



- We use a "random" QW dynamic in OAM and polarization, implemented via qplates, as *quantum reservoir*.
- With this we demonstrate the feasibility, efficiency, and *robustness* of reconstructing observables via QELMs.

Photonic experimental implementation



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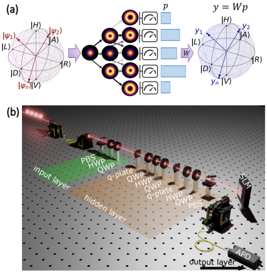
Photonic experimental implementation



- We benchmark the accuracy of QELM-based reconstruction on the best alternative method, which in this case would be a shadow-tomography-based approach.
- With this we demonstrate the feasibility, efficiency, and *robustness* of reconstructing observables via QELMs.

Summary

- QELMs offer a flexible architecture to extract features of input states via uncharacterised dynamics, trading knowledge of the dynamic with knowledge of a characterised training dataset.
- We demonstrate this with a photonic quantum walk architecture in OAM and polarisation.
- Our framework also applies to QRCs, by suitably redefining what the "effective POVM" is. Memory capacity of the dynamic will then determine nonlinearity of possible targets.
- Shadow tomography on measurement frames is a flexible framework to understand how to extract features from data in the general case, and also gives insight into how QELMs work.