ENVIRONMENT

Exploring dynamical quantum phase transitions and work extraction in open quantum systems using matrix product states

Grazia Di Bello

Environment induced dynamical quantum phase transitions

I

Open quantum batteries vs quantum phase transitions

II

DI BELLO, G., et al., *Environment induced dynamical quantum phase transitions in two-qubit Rabi model,* Commun. Phys., 2024, 7.1: 364.

DI BELLO, G., et al., *Local ergotropy and its fluctuations across a dissipative quantum phase transition*, Quantum Sci. Technol., 2024, 10: 015049.

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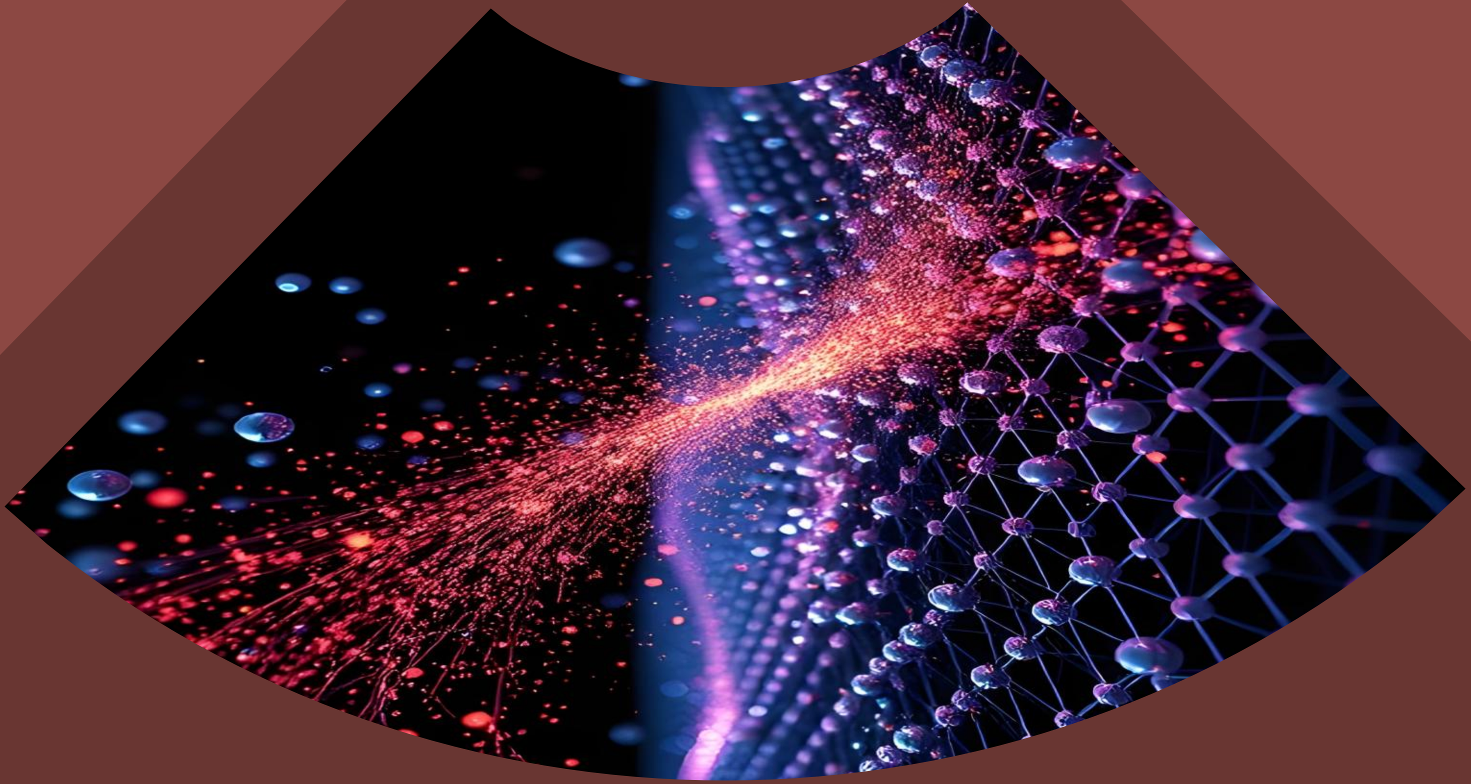
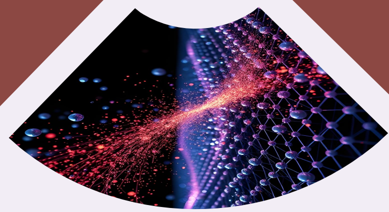
ICFO

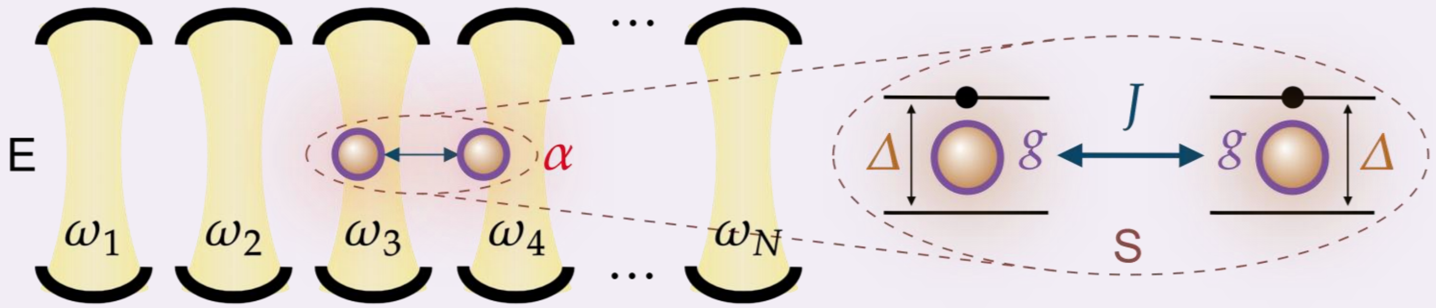


Environment induced dynamical quantum phase transitions

Dynamical quantum phase transitions in the two-qubit open quantum Rabi model

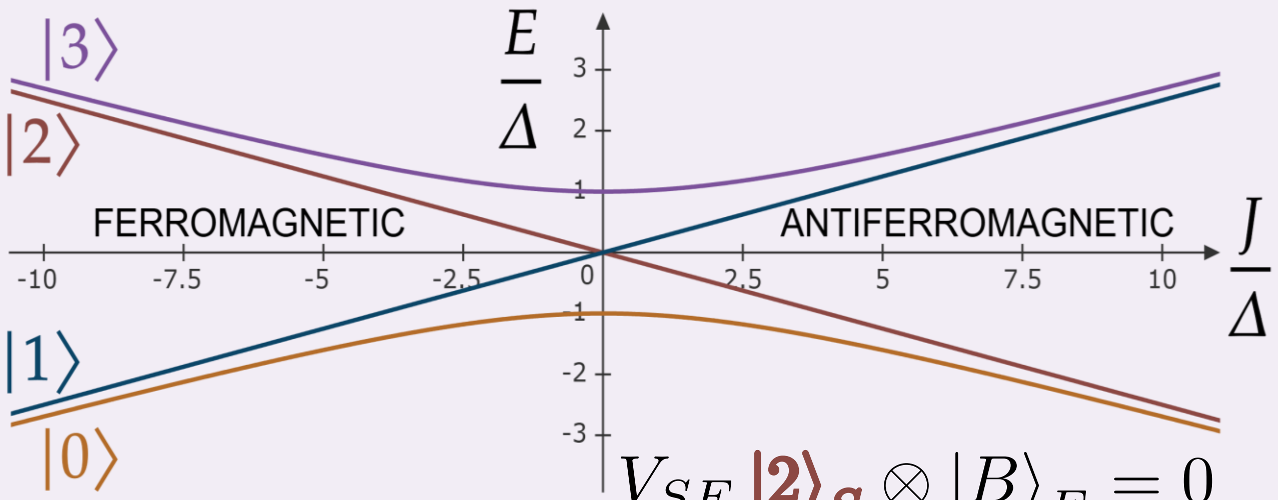


THE DISSIPATIVE QUANTUM RABI MODEL WITH TWO QUBITS



OPEN TWO-QUBIT RABI MODEL

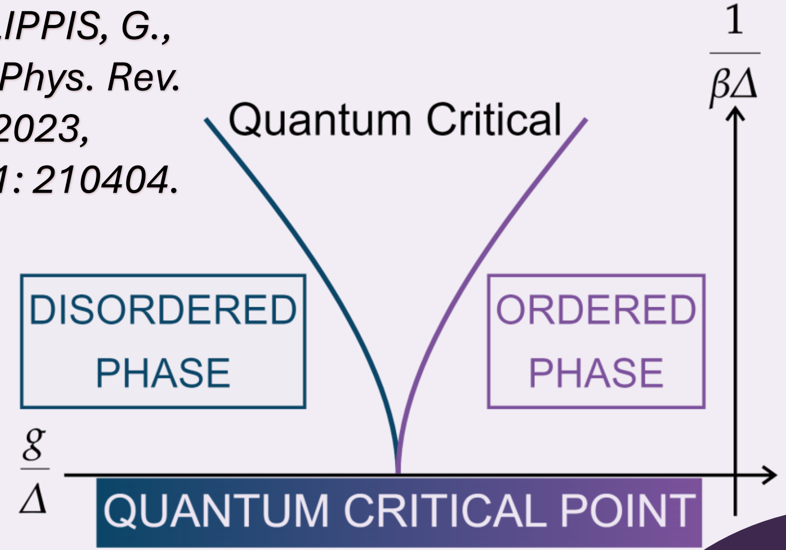
Decoherence-free state (DFS)



$$V_{SE}\,|2\rangle_S \otimes |B\rangle_E = 0$$

BKT thermodynamic QPT at zero T

DE FILIPPIS, G., et al., Phys. Rev. Lett., 2023, 130.21: 210404.

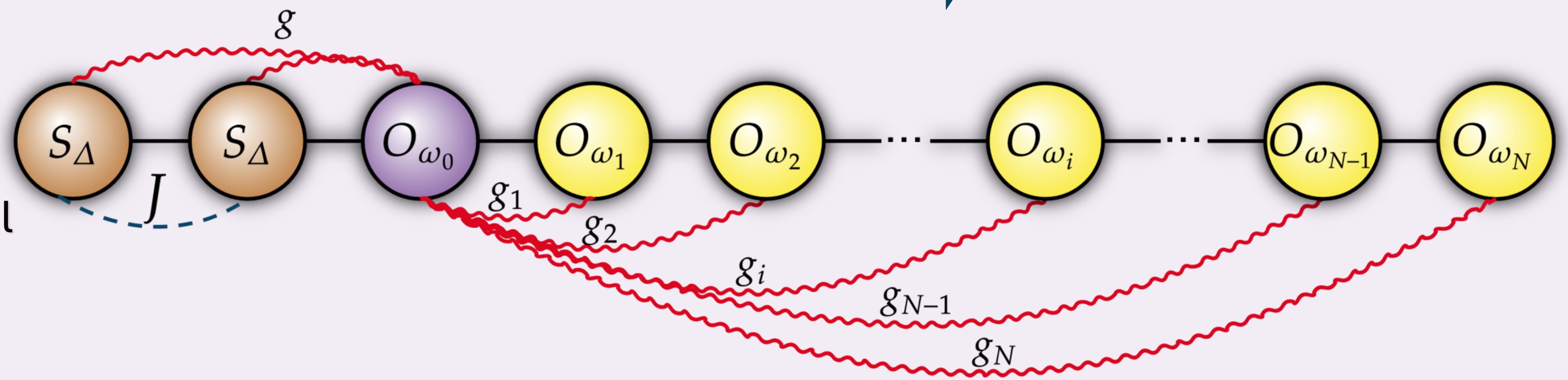


MPS TO REPRESENT THE SYSTEM STATE

- No exact analytical solution
- Mean-field/perturbation theory fails at strong coupling
- Exact diagonalization: $O(\exp(L))$

Accurate and efficient numerical simulations: **MPS**

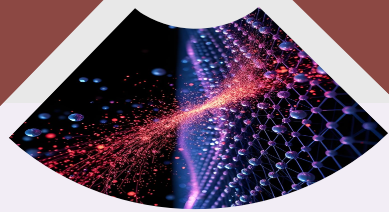
Example: two-qubit Rabi model



Wavefunction of the universe: SUBSYSTEM + BATH

$$|\psi(0)\rangle = (\alpha\,|\uparrow\uparrow\rangle + \beta\,|\uparrow\downarrow\rangle + \gamma\,|\downarrow\uparrow\rangle + \delta\,|\downarrow\downarrow\rangle) \otimes \sum_{n=0}^{n_{osc}} c_n\,|n\rangle \otimes |n_1, \ldots, n_N\rangle$$

How to unveil DQPTs and relate to QPT?



Numerical techniques

In equilibrium

- Worldline Monte Carlo (MC)
- Density Matrix Renormalization Group (DMRG)

Out of equilibrium

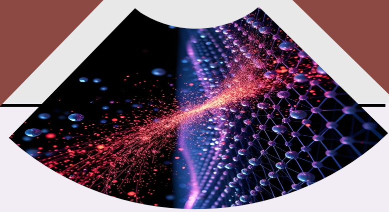
- Time-Dependent Variational Principle (TDVP)

ED: exp(L) →

MPS: LD^2d

poly(D,L)

MPS-based methods



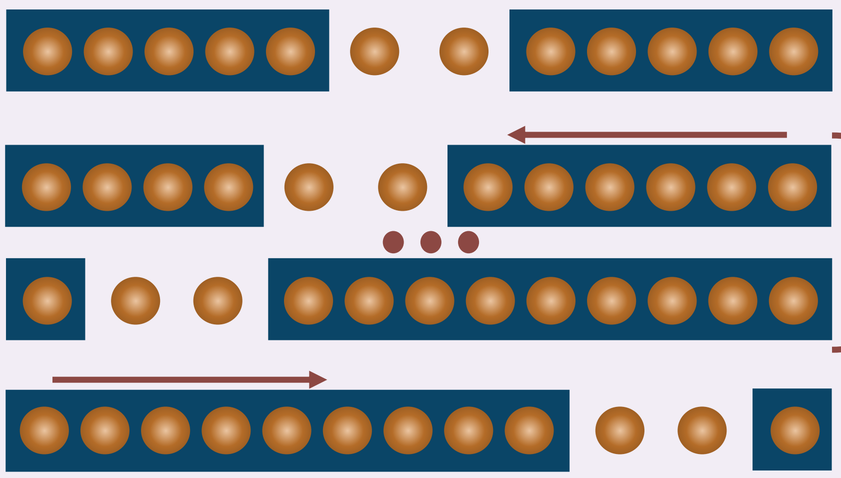
Density matrix renormalization group (DMRG) → **Ground state search**

SCHOLLWOCK, U., Ann. Phys., 2011, 326.1: 96192.

Time dependent variational principle (TDVP) → **Time-dependent evolution**

HAEGEMAN, J., et al., Phys. Rev. Lett., 2011, 107.7: 070601

Finite-size ≡ 2-site variational optimization

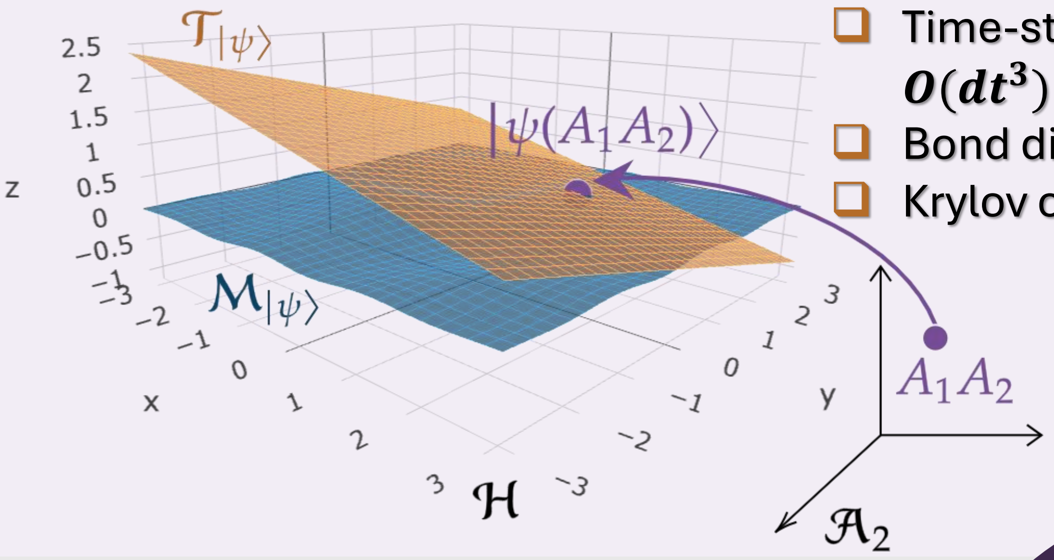


initial infinite-size warm-up

half sweep

stop if energy change below a tolerance

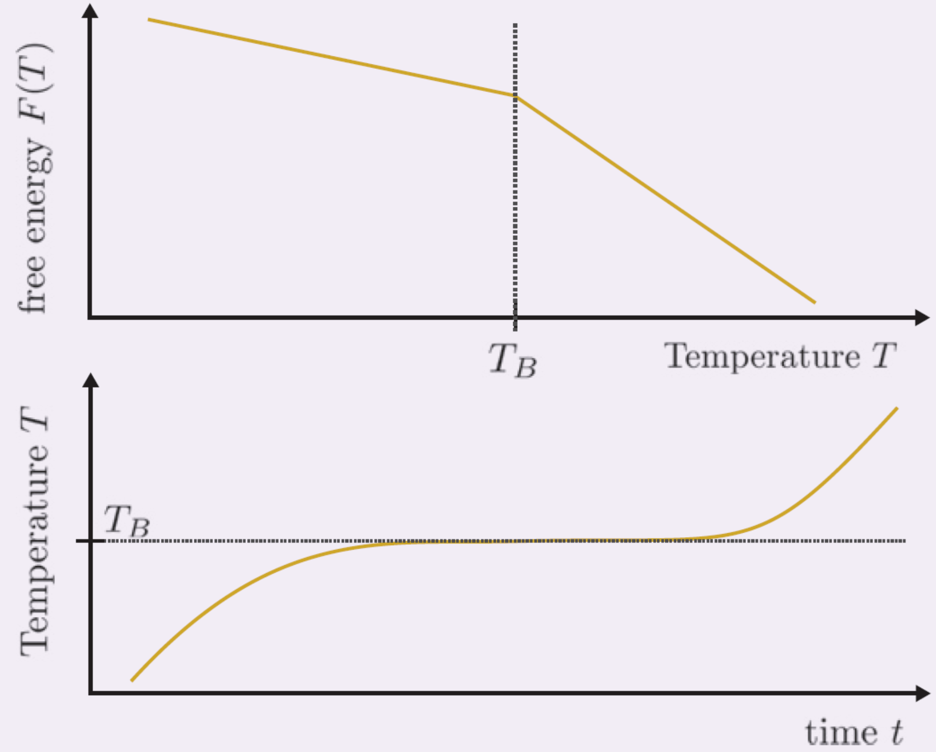
$$-i\frac{d}{dt}|\psi\rangle = \hat{P}_{T_{|\psi\rangle}} H |\psi\rangle$$

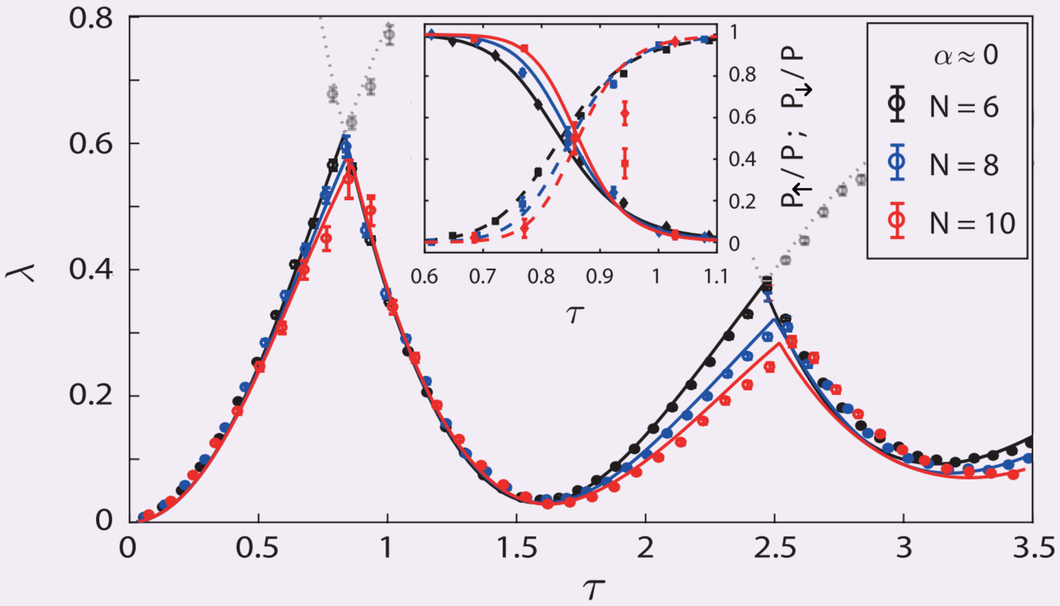


- Time-step error $O(dt^3)$
- Bond dimension
- Krylov cutoff

DYNAMICAL QUANTUM PHASE TRANSITIONS IN THE TWO-QUBIT QRM

HEYL, M., Europhys. Lett., 2019, 125.2: 26001.





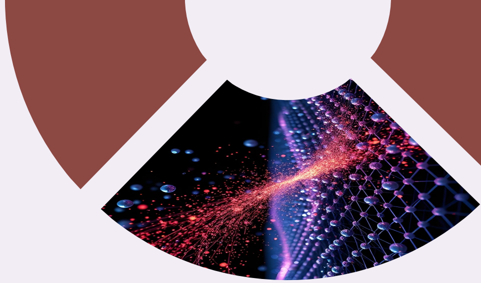
JURCEVIC, P. et al., Phys. Rev. Lett., 2017, 119: 080501.

$$|\psi_0\rangle \longrightarrow |\psi_0(t)\rangle = e^{-\frac{i}{\hbar}Ht}|\psi_0\rangle$$

Quantum quench $H_0 \longrightarrow H$

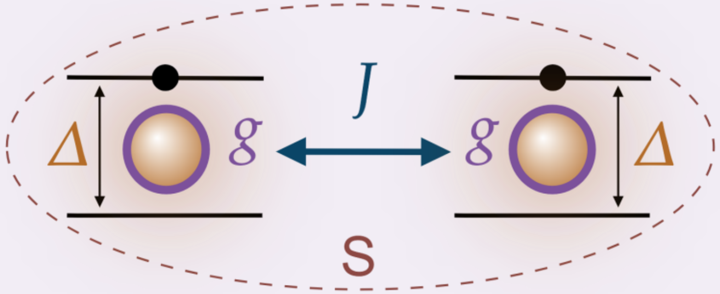
$$\mathcal{L}(t) = \left|\langle\psi_0| e^{-\frac{i}{\hbar}Ht} |\psi_0\rangle\right|^2 = e^{-\mathcal{N}\lambda(t)}$$

Loschmidt echo probability



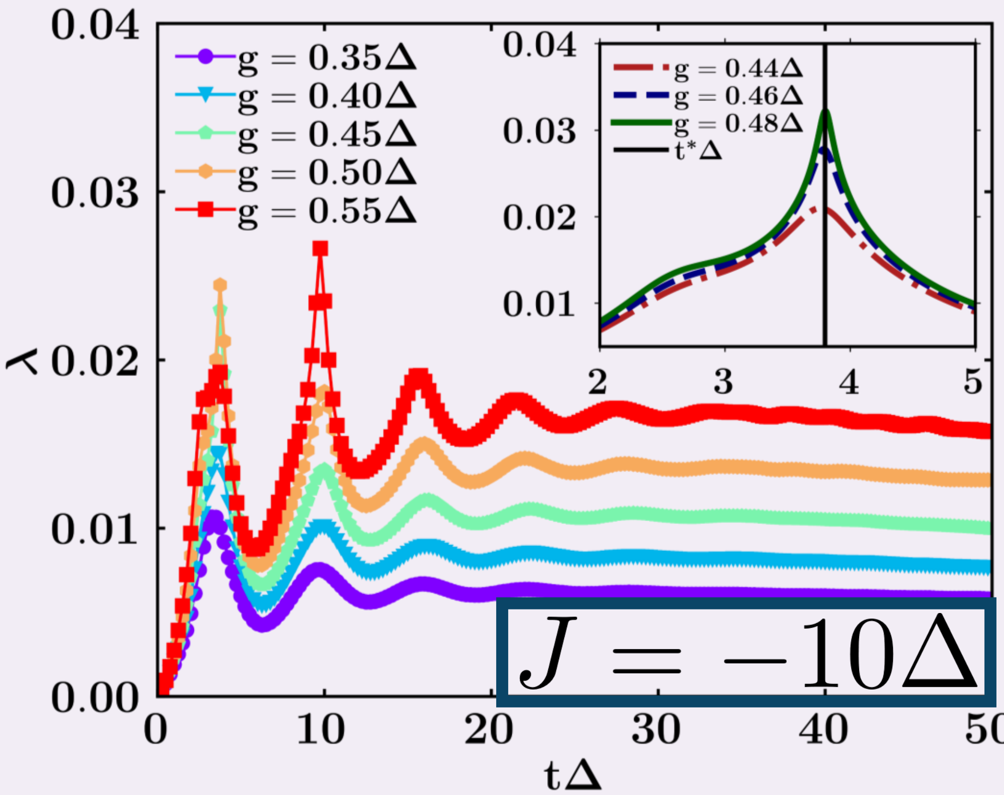
ENVIRONMENT INDUCED DYNAMICAL QUANTUM PHASE TRANSITIONS IN TWO-QUBIT RABI MODEL

DI BELLO, G., et al., Commun. Phys., 2024, 7.1: 364.



$$\begin{cases} H\big|_{g=0} \equiv H_0 \\ H\big|_{g\neq 0} \equiv H \end{cases}$$

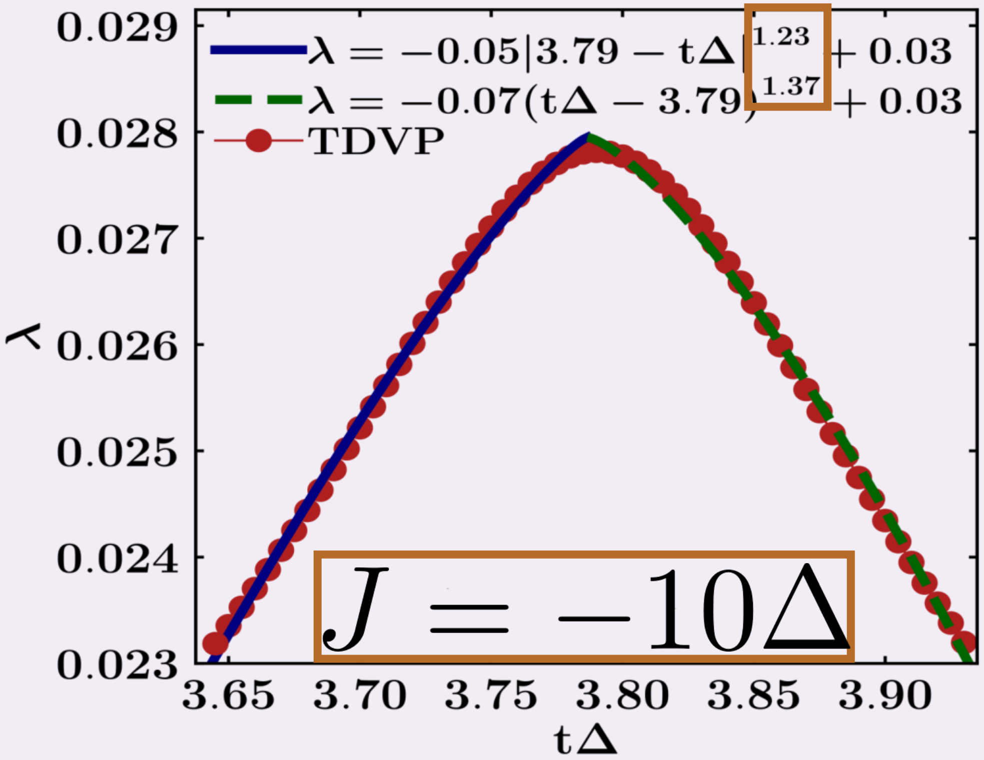
Our TDVP results: Loschmidt echo rate function

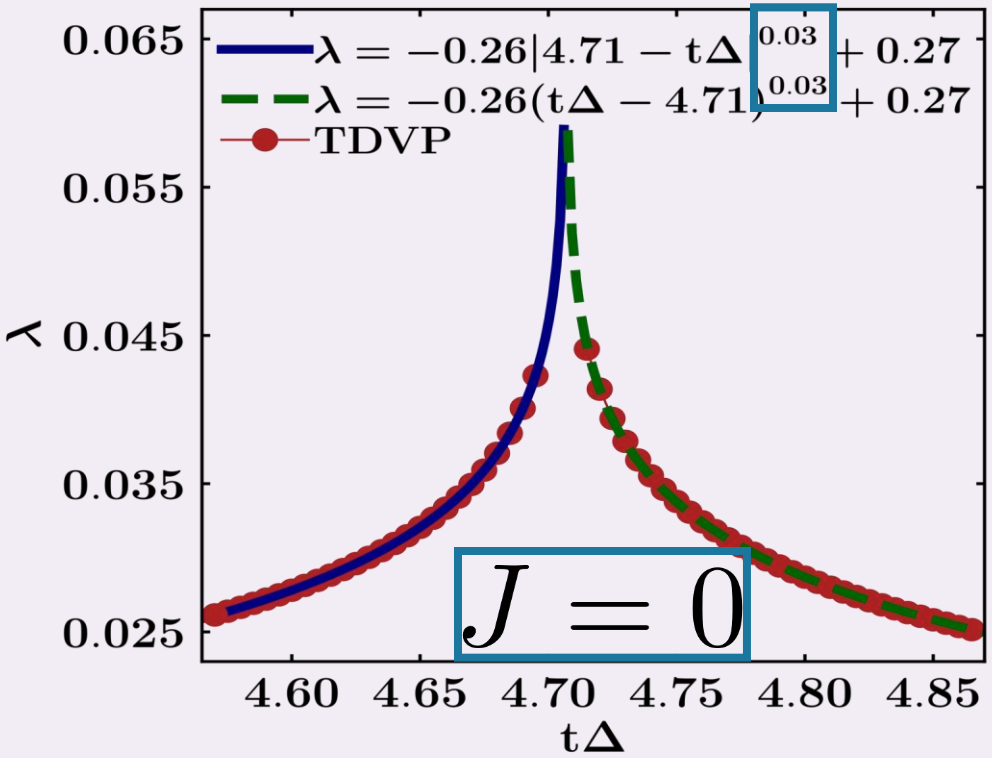


Rate function kinks towards the DQPT for the same parameters range as QPT

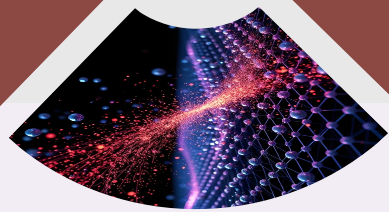
Our TDVP results: DQPTs critical exponents

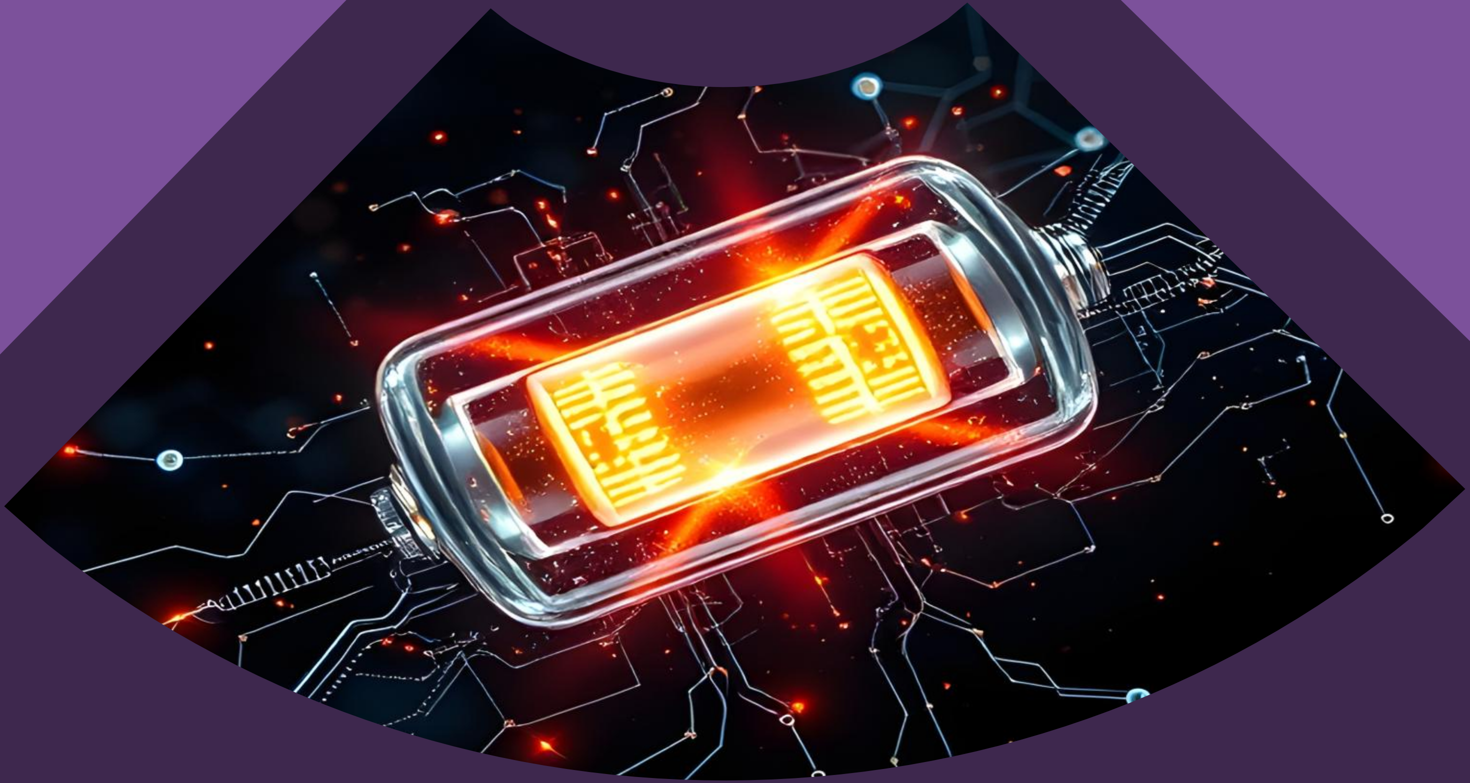
HEYL, M., Phys. Rev. Lett., 2015, 115: 140602.





1) Range of predominating interactions:	SHORT	LONG
2) Initial interactions and entanglement:	YES	NO
3) Critical exponent:	QUASI-LINEAR	SUB-LINEAR





Open quantum batteries vs quantum phase transitions

Ergotropy and its local formulation in many-body systems

WORK EXTRACTION PROTOCOLS

ERGOTROPY

Maximum amount of extractable work

$$\mathcal{E} = \max_{U \in \mathcal{U}(d)} \left\{ \mathrm{tr}(H\rho) - \mathrm{tr}\left(U\rho U^\dagger H\right) \right\}$$

ALLAHVERDYAN, A. E., et al., Europhys. Lett., 2004, 67.4: 565.

Switch-off

Turning off the bath interaction

$$\mathcal{E}_{so} = \mathcal{E}_{sub} - \Delta_{so}$$

KOSUKE, I., et al., 2017, arXiv:1711.02322.

Local

Acting only on the two qubits

$$\mathcal{E}_S = \max_{U_S \in \mathcal{U}(d_S)} \left\{ \mathrm{tr}(H_{SE}\rho_{SE}) - \mathrm{tr}\left(U_S \rho_{SE} U_S^\dagger H_{SE}\right) \right\}$$

SALVIA, R., et al., Phys. Rev. A, 2023, 107.2: 01240.

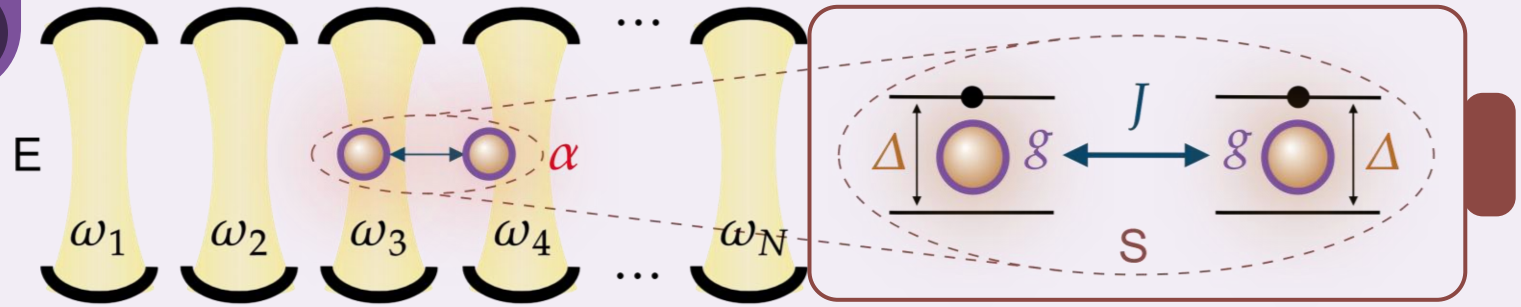
Fluctuations

Quasi-probability distribution of work

$$\sigma^2(t) = \langle w^2 \rangle - \langle w \rangle^2$$
$$= \mathrm{tr}\left[(H' - H)^2 \rho_{SE}(t)\right] - \left\{\mathrm{tr}[(H' - H)\rho_{SE}(t)]\right\}^2$$

FRANCICA, G., Phys. Rev. E, 2022, 105.1: 014101.

Our method to charge and discharge

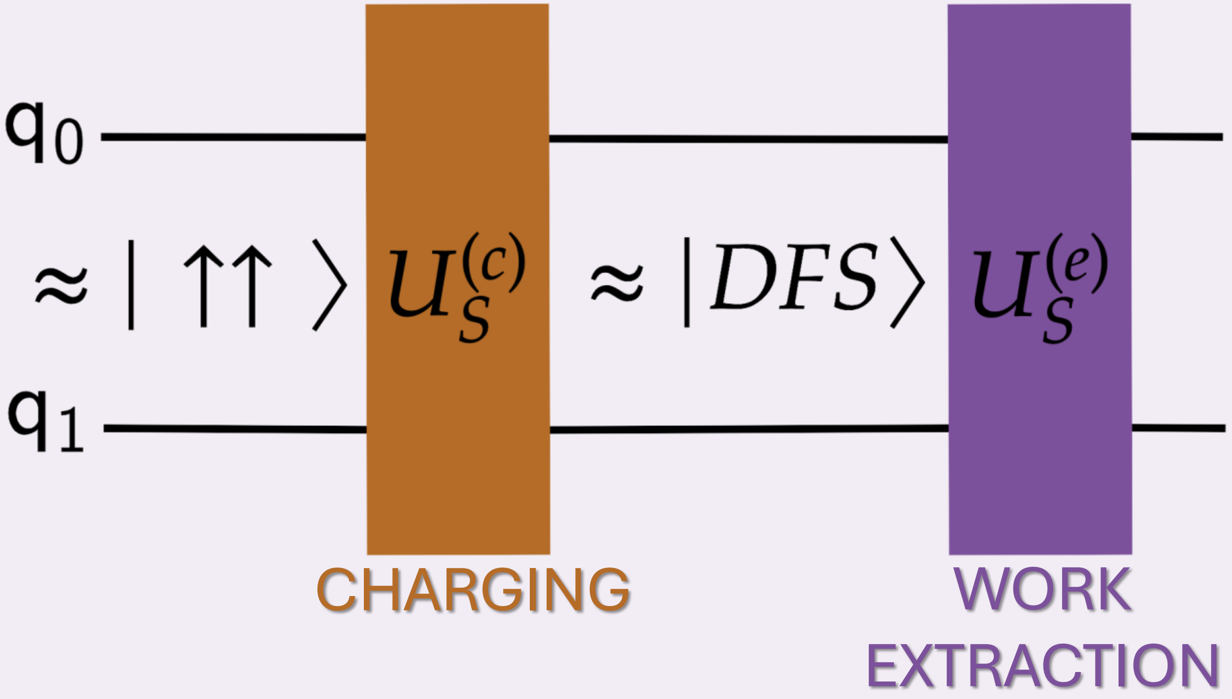


DI BELLO, G., et al., Quantum Sci. Technol., 2024.

OPEN QUANTUM BATTERY

$$|\psi(0)\rangle = |GS\rangle_{DMRG}$$

Initial **ground state** of entire system



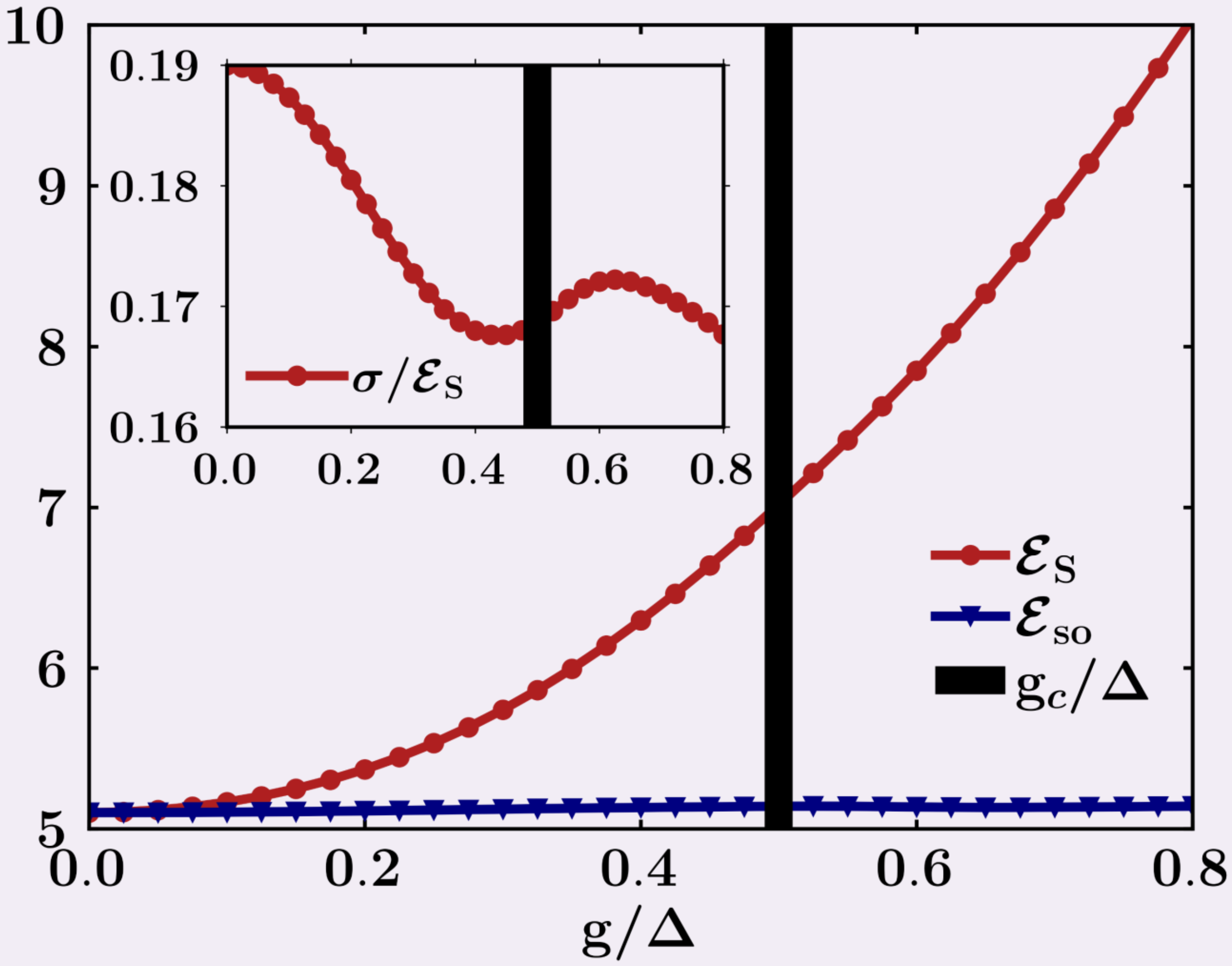
"Agnostic" protocol

apply $U_S^{(e)} = U_S^{(c)\dagger}$

Bayesian optimization (physically inspired)

apply $U_S^{(e)} = U_S'(\bar{\theta}, \bar{\phi})\, U_S^{(c)\dagger}$

Our DMRG results just after charging

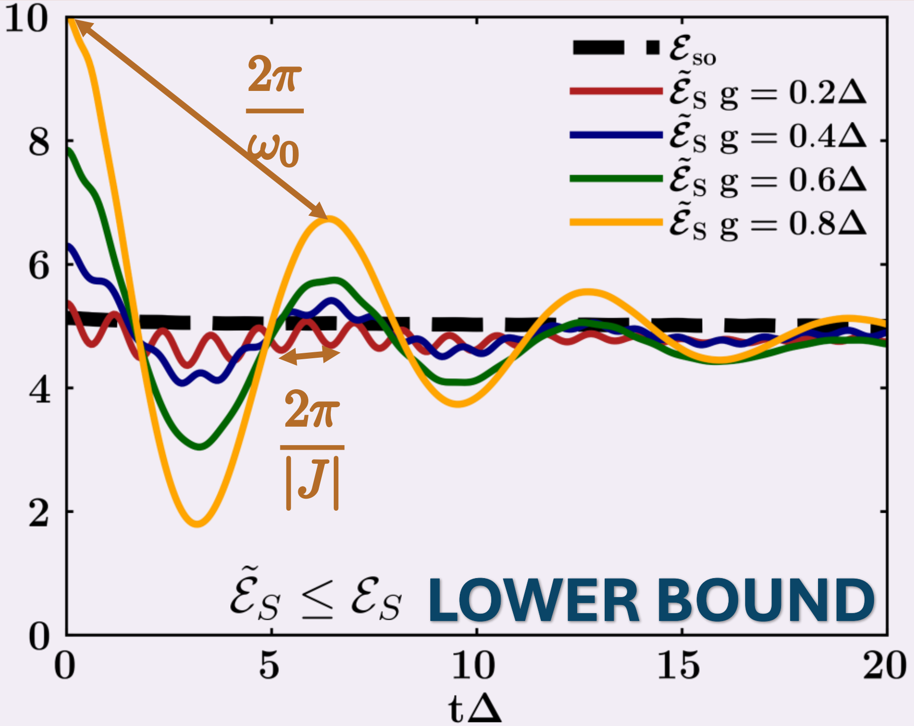


LOCAL ERGOTROPY $\mathcal{E}_S = E_c - E_{GS}$

- Coincides with the total
- Doubles SO (≈ maximum energy extractable from qubits)
- Bath is not detrimental
- Growth depends on QPT
- Fluctuations sensitivity to QPT

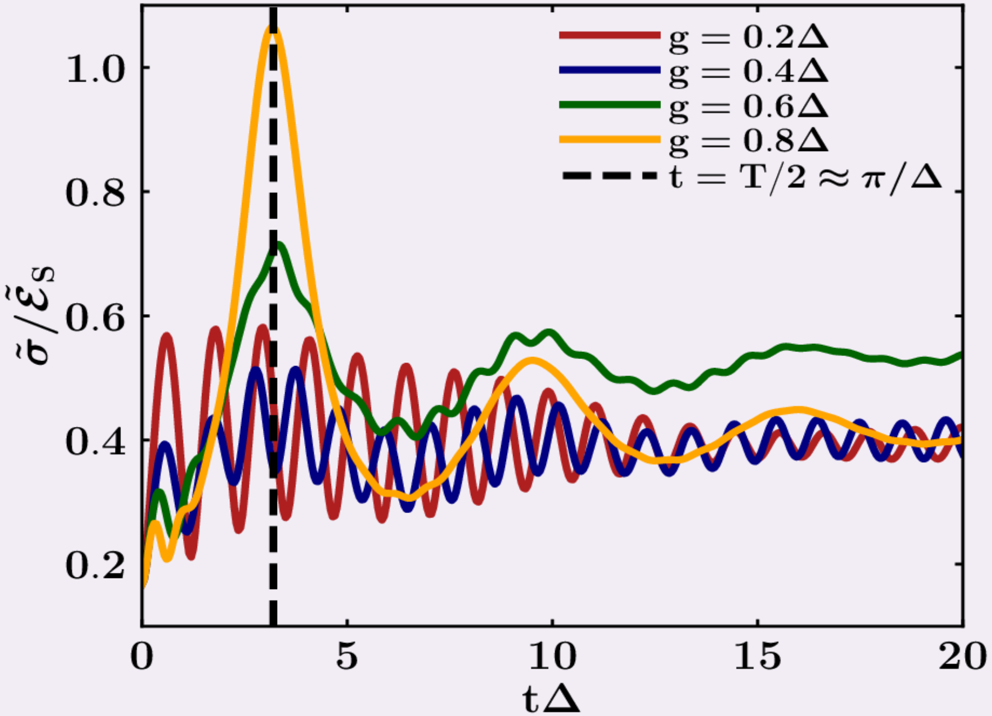
Our TDVP results: “agnostic” protocol

Local ergotropy



Non-zero ergotropy behaviors mark QPT due to in-phase and counter phase bath oscillators, with super-oscillations at $|J|$

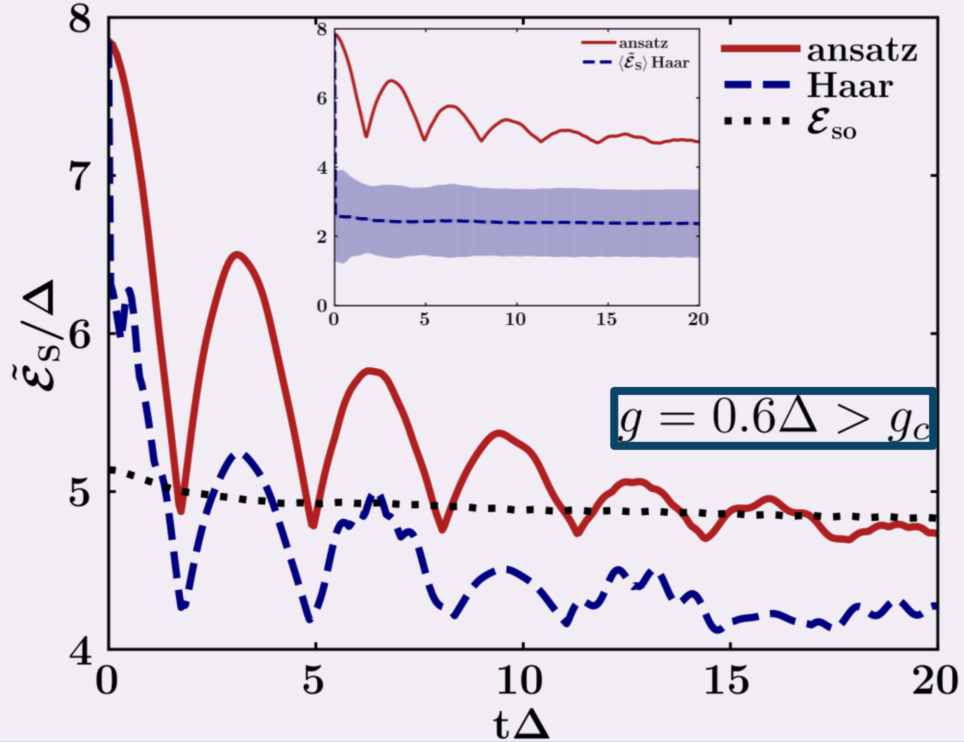
Relative fluctuations



Fluctuations’ different behaviors and first maximum mark QPT, with maxima corresponding to minima of ergotropy

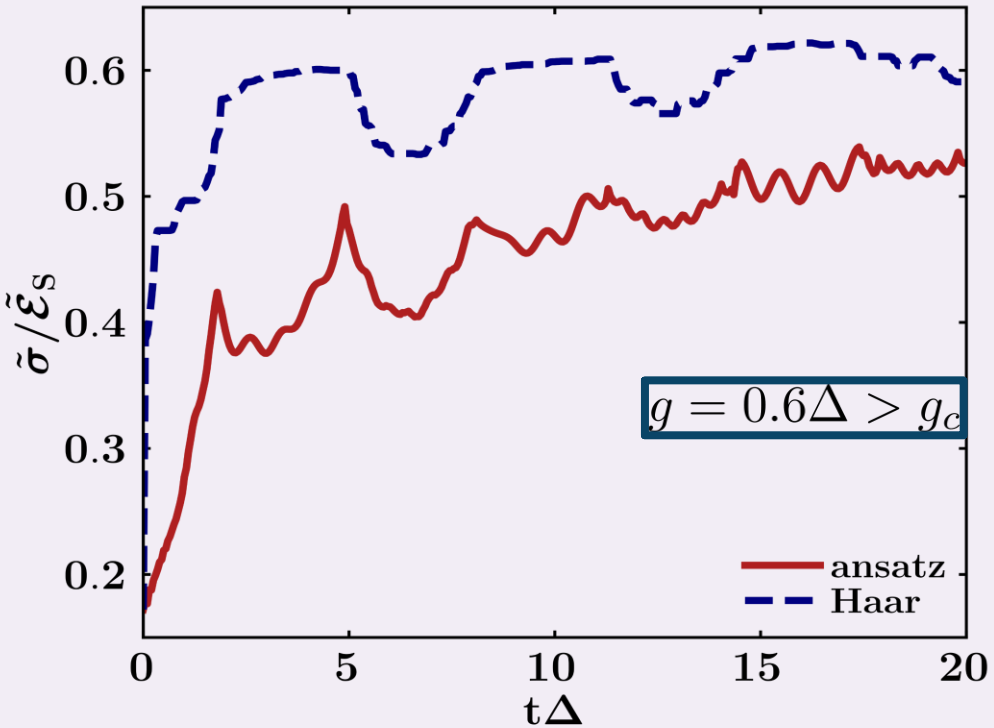
Our TDVP results: Bayesian optimization

Local ergotropy



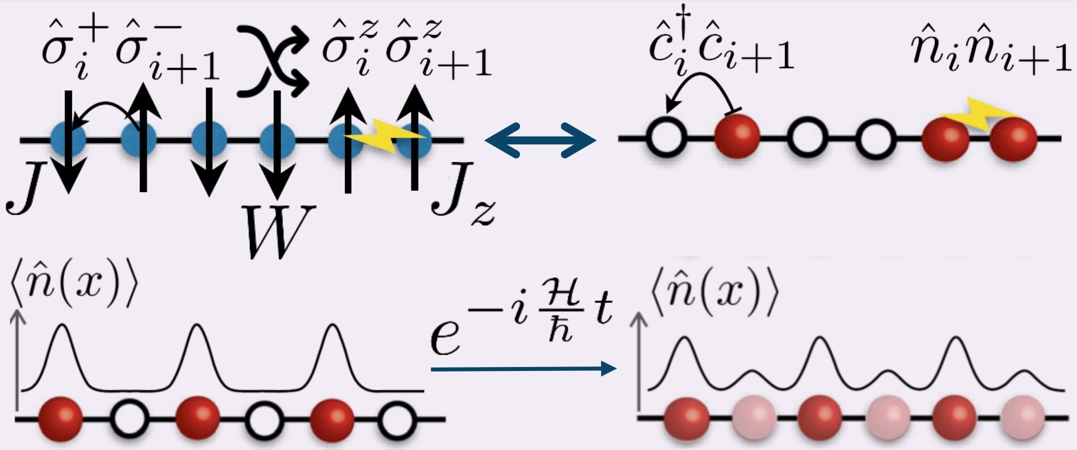
Rephasing bath oscillators makes the ansatz outperform Haar distributed unitaries and reaches SO at long times

Relative fluctuations



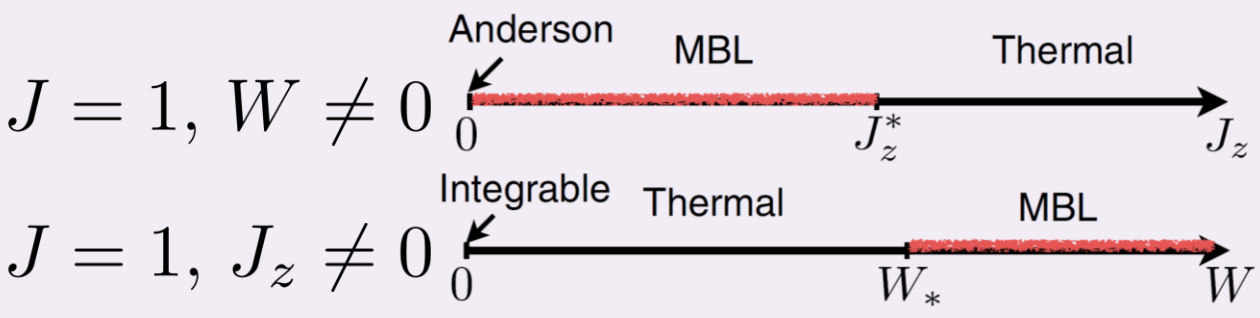
More contained fluctuations make the protocol more precise than the agnostic and the Haar ones

LOCAL ERGOTROPY IN MANY-BODY LOCALIZED SYSTEMS



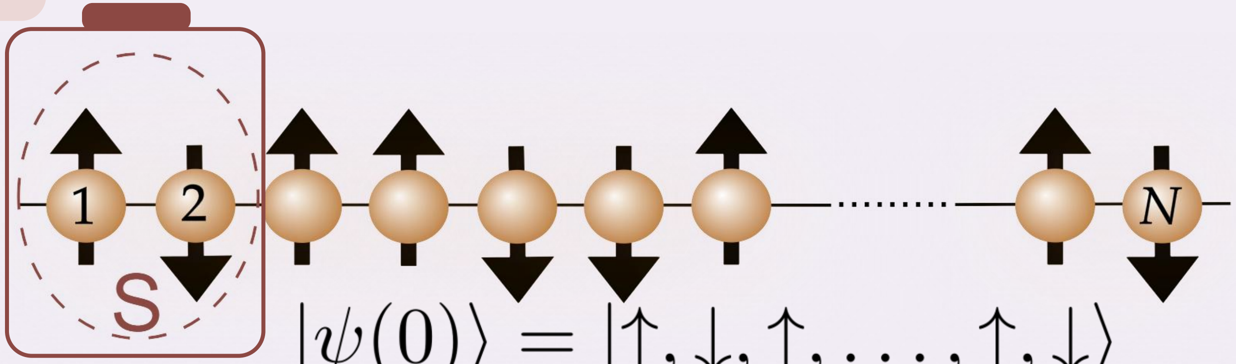
Many-body localized state

Retains memory of initial pure state



ABANIN, D. A., et al., Rev. Mod. Phys., 2019, 91.2: 021001.

OPEN QUANTUM BATTERY

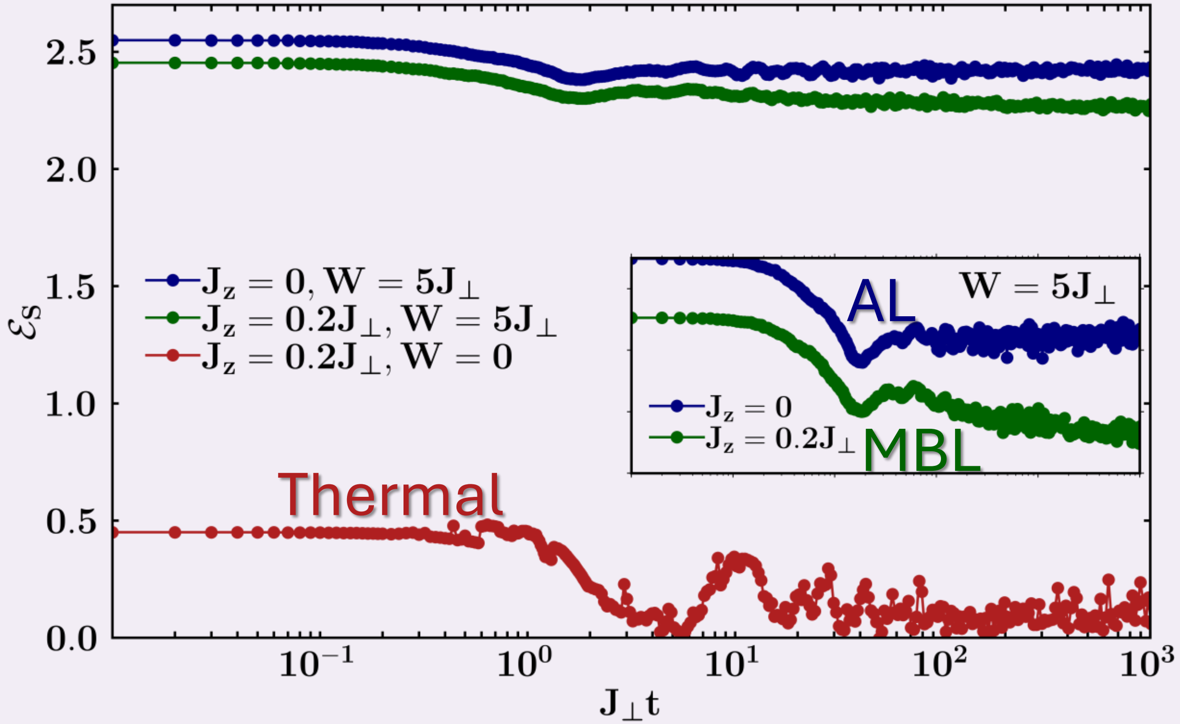


$$|\psi(0)\rangle = |\uparrow, \downarrow, \uparrow, \ldots, \uparrow, \downarrow\rangle$$

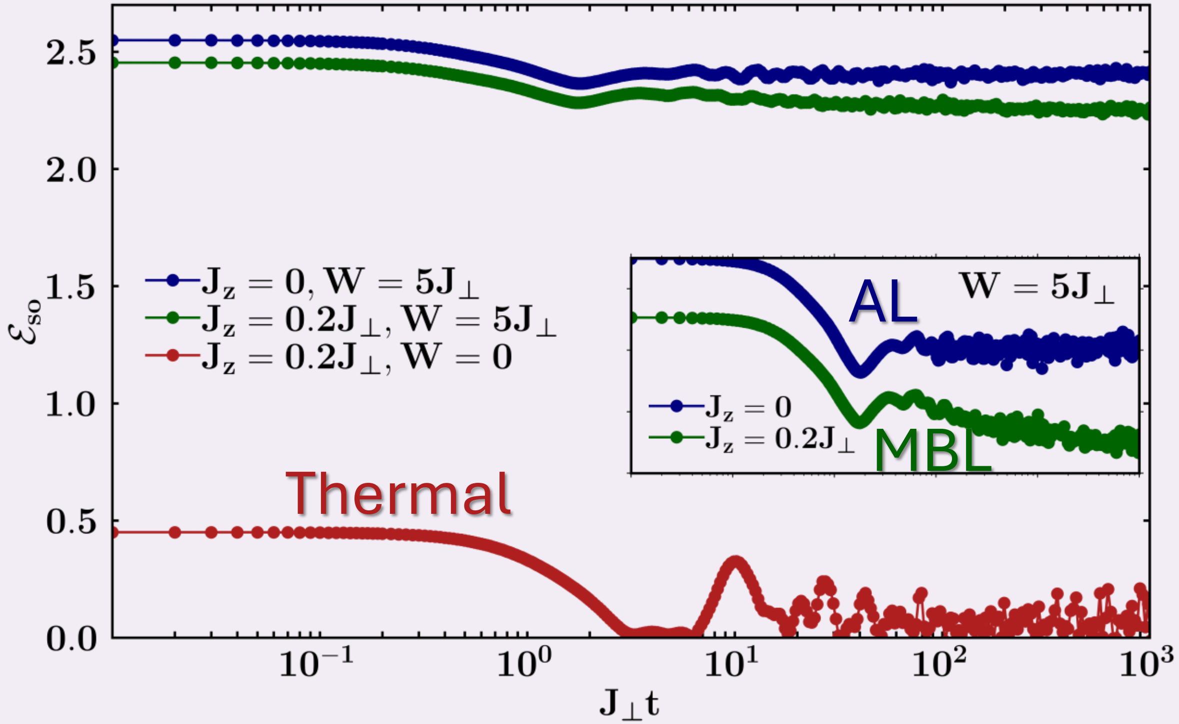
Initial **Néel** state

Our TDVP results: ergotropy

Local



Switch-off



Ergotropy saturates (AL), logarithmically decreases (MBL), rapidly decays to zero (thermal). Some energy gets “trapped” in the interactions, creating correlations, but the logarithmic entanglement allows to extract a significant amount of work.

KEY TAKEAWAYS

Bath-induced **DQPTs** depending on **interactions** and **entanglement**

Local ergotropy and **fluctuations** mark the QPT and MBL-AL-thermal transition

Experimentally reachable DQPTs and **criticality-enhanced** quantum sensing

Protocol for work extraction from similar systems and **markers** of different transitions

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WHAT'S NEXT? ▶▶❙